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A POLARIZATION RESPONSIVE SYSTEM
FOR MICROWAVES

THESIS

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A POLARIZATION RESPONSIVE SYSTEM FOR MICROWAVES

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THESIS

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Table of Contents

List of Figures

List of Tables

AFIT/GE/EE/80D-32

Abstract

Responsive polarization methods and concepts were studied in terms of general system requirements. A primary goal was fast response. Matrix methods were found appropriate for describing the polarization states and modifications thereto.

A 9.3 GHz experiment, of one promising concept, was constructed. Stokes parameters were measured, modified and converted to Jones parameters for response.

Basic feasibility was established for the hybrid Tee circuit arrangement which was developed for the thesis experiment. The results were as predicted by theory.

A POLARIZATION RESPONSIVE SYSTEM

FOR MICROWAVES

I Introduction

Problem

This report investigates the theory and conceptual application of polarization principles to a microwave system for receiving incident plane electromagnetic waves, determining their polarization characteristics, then retransmitting the waves with modified polarization characteristics in the reverse direction. The specific problem is to identify the polarization characteristics of an incoming electromagnetic wave in a mathematically tractable parameter matrix so that responsive replies can be formulated.

Background

Previous polarization studies by Kraus (1966), Shurcliff (1962), Collett (1971), Cornbleet (1976), and others give the basic tools of polarization analyses which include Stokes parameters, coherency matrices, Poincare sphere, and Jones and Mueller matrices. Two polarization states are of special practical interest: the matched polarization or maximum power transfer case and the orthogonal or "null" polarization case which in theory gives zero received signal. In a practical system, however, every antenna system has residual response in its "null" polarization, although it may be many tens of dB below the matched case. Such null polarization response is usually increased by any radome structure over the antenna or by scattering from nearby objects (including the earth's surface). Radar interference (intentional jamming) can be created by responding with cross-polarized signals of sufficient amplitude to compete with the skin return of the radar target.

Rapid polarization change, either intentional or that caused by natural factors, can disrupt the jamming process if the jammer is too slow to follow the changes; therefore, a desirable system capability is response within a fraction of a microsecond.

Assumptions

All of the concepts presented in this study are inherently broadband, since the phenomenon does not depend on tuned or resonant circuits. In an actual system, each microwave component, antenna radome, etc. will have a certain bandwidth limitation which becomes significant once a specific design application is identified. For the purpose of this study, broadbanding capability is assumed and feasibility is established at only one frequency in the I band microwave region.

Scope

The experimental portion of this study investigates only the microwave circuitry. It excludes the digital processing portion of the system which is recommended for future experimental verification, perhaps by a future graduate student.

An operational system would likely be integrated into an aircraft, covered by a radome, and employ beam steering antennas. This study, however, is limited to just the polarization aspects in a laboratory environment. This work is intended as the first conceptual step, in the chain of events that could eventually lead to diverse kinds of operational systems, i.e., communication (using polarization states), radar jamming with other than the working polarization, or echo reduction by retransmitting a coherent out-of-phase replica of the signal scattered from a vehicle.

General Approach

The problem was attacked by seeking out existing polarization mathematical tools, many of which originated from optics. Matrix methods, including Jones, Stokes, Mueller, Maxwell, and coherency matrices, give the capability to describe and operate on the state of polarization of a wave. These methods were sought for their useful application to the responsive system. Several candidate concepts with different implementations were formulated. Then an experimental system concept was chosen which senses the Stokes vector parameters, manipulates them through Mueller matrix multiplication, followed by a Stokes to Jones conversion and finally transmits through a Jones vector transmitter. Experimental data was measured and reduced to show that the two-antenna Stokes receiver works as the concept predicts. The Jones transmitter is a direct hardware implementation of the elements of the Jones vector.

Sequence of Presentation

The thesis material is presented in the following sequence; first system theory, then system concepts, applications, equipment, procedure, results and finally conclusions. The Appendix contains raw data, repeatability measurements, error analysis, a computer program listing for polarization matrix multiplication and methods for matrix or vector conversion.

II System Theory

Matrix Theory

A concise mathematical formulation is needed to describe the electromagnetic wave polarization because the number of simultaneous parameters soon becomes unwieldy. Matrix notation was recognized as a powerful tool for such formulation by the scientists working in optics (Jones 1941, Shurcliff 1962). They were concerned with passage of light through, or reflected from, various materials. Some of the materials such as the man made sheet polarizers were developed for specific characteristics while other materials were natural substances. The number of polarization modifying materials was limited to only a few cases.

In the microwave system concept of this thesis, the states of polarization are uncountably large and it is possible to electronically alter the polarization by an uncountably large number of electronic modifiers.

The matrix calculus used in optics provides a straightforward, systematic way to deal with these large numbers of polarization states.

In this section the Stokes vector, Mueller matrix, Jones vector, Jones matrix, and the coherency matrix will be described in that order.

Stokes Vector

Four quantities (Stokes parameters) completely describe the intensity and polarization of an electromagnetic wave. They are proportional to time averages or energies associated with different components of the field.

The four Stokes parameters, which are called I,Q,U,V by Kraus or I,M,C,S by others, comprise a column vector. The first parameter is intensity of polarization or ratio of polarized to unpolarized energy. The second "Q" or "M" parameter is horizontal preference. "U" or "C" is preference for 45° and the fourth parameter "V" or "S" is left circular preference. Negative parameter values are orthogonal to those listed.

Measured energies to represent the four parameters result from averaging each over a long enough time to encompass at least one-half of the radio frequency sinusoidal period. Sensing can be accomplished by six separate antennas whose output power is combined as follows to create the normalized Stokes Parameters (fully polarized case):

$$I = \frac{P_R+P_L}{P_R+P_L} = \frac{P_H+P_V}{P_H+P_V} = \frac{P_{45}+P_{-45}}{P_{45}+P_{-45}} = 1$$

$$M = \frac{P_H-P_V}{P_H+P_V}$$

$$C = \frac{P_{45}-P_{-45}}{P_{45}+P_{45}}$$

$$S = \frac{P_L-P_R}{P_R+P_L}$$

where

P_R = power from a right circularly polarized antenna

P_L = power from a left circularly polarized antenna

P_H = power from a horizontally polarized antenna

P_V = power from a vertically polarized antenna

$P_{45°}$ = power from a slant 45° polarized antenna

$P_{-45°}$ = power from a slant -45° polarized antenna

Instead of the six separate antennas, equivalent power responses for the six cases could be obtained from two orthogonal antennas, such as horizontal and vertical, by appropriately phasing their RF outputs and summing prior to time averaging.

Mueller Matrices

The Mueller matrix is a 4 by 4. It is useful in describing the change in the state of polarization as a wave passes through a polarizing medium. It encompasses incoherent and partially polarized waves. The Mueller matrix contains 16 constants, only seven of which are independent (Shurcliff, 1962).

If a wave is initially represented by the Stokes column vector G with components I, M, C, S then the scattered wave is

$$\underset{(4\text{x}1)}{G'} = \underset{(4\text{x}4)}{[\,M\,]}\;\underset{(4\text{x}1)}{[\,G\,]}$$

Absolute Phase (referenced to some point of origination of the wave) is not contained in the 4x4. One more coefficient would be necessary to define it.

The Mueller matrix of any device at a non-principal azimuth (azimuth refers to rotation angle θ of the fast axis about a line in the direction of propagation) can be factored into three matrices, one of which (the central one) describes the device.

$$P_\theta = S(\theta)P_0S(-\theta)$$

The rotation matrices are

$$S(\theta) = \begin{bmatrix} C_1 & S_1 \\ -S_1 & C_1 \end{bmatrix} \qquad S(-\theta) = \begin{bmatrix} C_1 & -S_1 \\ S_1 & C_1 \end{bmatrix} \qquad \text{where} \quad \begin{array}{l} C_1 = \cos\theta \\ S_1 = \sin\theta \end{array}$$

Jones Vector

The Jones vector (Jones, 1941) represents the state of polarization by a vector of two components, each containing amplitude and phase information. The state of polarization is completely defined by the amplitudes and phases of the x and y components of the electric vector of the wave at a fixed point along the z axis. The vector elements can be derived from two orthogonally polarized antennas. In general the Jones vector can be stated as

$$A \begin{bmatrix} \cos\theta \; e^{-j\delta/2} \\ \sin\theta \; e^{j\delta/2} \end{bmatrix} = \begin{bmatrix} A_x e^{-j\delta/2} \\ A_y e^{j\delta/2} \end{bmatrix}$$

where θ is the angle whose tangent is $\frac{A_y}{A_x}$ and δ is the phase angle between the y and x components.

The method is based on the idea that any wave can be represented as the resultant of the coherent addition of two orthogonal linearily polarized waves with appropriate amplitudes and phases. It is well suited to problems involving a large number of similar devices arranged in series in a regular manner; however is not applicable to devices that have depolarizing tendencies (i.e., only part of the wave energy is coherently polarized). The Jones Method is derived directly from electromagnetic theory, while the Mueller calculus is based on a phenomenonalogical foundation. It permits one to preserve absolute phase information; therefore, it is useful for problems involving the combining of two or more coherent signals.

Jones Matrix

R. Clark Jones used a 2 by 2 matrix to describe polarization changes as light passes through an optical system. The four elements of the matrix contain eight independent constants (four amplitude and four phase constants). The general form of the full Jones matrix is (Cornbleet, 1976, p 292-329):

$$\begin{bmatrix} \cos^2\theta\, e^{j\delta/2} + \sin^2\theta\, e^{-j\delta/2} & 2j\cos\theta\,\sin\theta\,\sin(\delta/2) \\ 2j\cos\cdot\ \sin\gamma\sin(\emptyset/2) & \cos^2\theta\, e^{-j\delta/2} + \sin^2\theta\, e^{j\delta/2} \end{bmatrix}$$

where θ is the azimuth (rotation angle) of the fast axis and δ is the phase between y and x components introduced by the polarizer. Its advantage is a smaller matrix than Mueller (2x2 vs 4x4) while its disadvantage is that the matrix elements are complex and not applicable to systems that handle unpolarized electromagnetic waves.

Coherency Matrix

The coherency matrix is used to predict the power response W of an antenna. Let the coherency matrix of the antenna be (Kraus 1966):

$$Ar\ X \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

When the antenna is illuminated by a wave with polarization represented by the coherency matrix

$$S\ X \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

The power W is given by the trace of the product of the two coherency matrices

$$W = \mathrm{Tr} \left\{ \mathrm{Ar} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad x \quad S \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \right\}$$

where "Trace" is the sum of the diagonal elements.

The coherency matrices (suppressing normalization) are Jones representations and are obtained from the Stokes vector elements as follows.

$$S_{11} = \tfrac{1}{2}(I+M) \qquad S_{12} = \tfrac{1}{2}(C+jS) \qquad S_{21} = \tfrac{1}{2}(C-jS) \qquad S_{22} = \tfrac{1}{2}(I-M)$$

where $\begin{matrix} I \\ M \\ C \\ S \end{matrix}$ is the Stokes vector representing the illuminating wave.

Likewise:

$$A_{11} = \tfrac{1}{2}(A_0 + A_1) \qquad A_{12} = \tfrac{1}{2}(A_2 + jA_3) \qquad A_{21} = \tfrac{1}{2}(A_2 - jA_3) \qquad A_{22} = \tfrac{1}{2}(A_0 - A_1)$$

where $\begin{matrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{matrix}$

is the Stokes vector representing the wave that would be created by the antenna if it were transmitting. Of course the Jones representation excludes the unpolarized portion of the wave and therefore the above conversion is not completely general.

Polarization Mismatch

The polarization of an electromagnetic wave refers to the spatial orientation and relative phase of its orthogonal vector components, usually taken in terms of the electric field. Let these components be described by (Kraus 1966):

$$\vec{E}_x = \hat{x}\, E_x e^{j(\omega t-kz)}$$

$$\vec{E}_y = \hat{y}\, E_y e^{j(\omega t-kz)}$$

for a plane wave propagating in the z direction (where $E_x = A_x e^{j\delta x}$ and $E_y = A_y e^{j\delta y}$). Then at any fixed location on the z axis the real parts are $E_x = A_x \cos(\omega t+\delta_x)$ and $E_y = A_y \cos(\omega t+\delta_y)$.

Suppressing the time dependence results in the relation

$$\frac{E_x^2}{A_x^2} + \frac{E_y^2}{A_y^2} - \frac{2E_xE_y}{A_xA_y} \cos\delta = \sin^2\delta$$

where δ is the difference between the absolute phase δy and δx. This equation describes a general ellipse in the xy plane which becomes a circle when $\delta = \frac{\pi}{2}$. When $\delta = 0$ the ellipse degenerates to a line (See Fig 1).

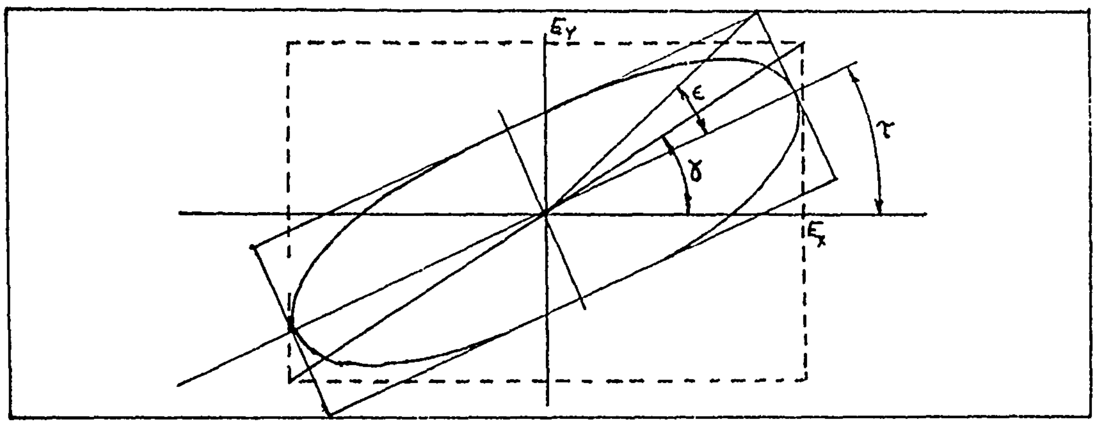

Fig 1. Relation of Ex, Ey and Angles ε, γ and τ to the Polarization Ellipse

The antenna can be thought of as an aperture which intercepts energy from a propagating electromagnetic plane wave. By reciprocity the same antenna will perform the transmitting function. The power density of an incoming plane wave polarized entirely in the x direction can be described by its Poynting vector $\vec{P}$ watts per meter^2 with magnitude $|E_x|$ times $|H_y|$ and direction $\hat{z} = \hat{x} \times \hat{y}$.

The power W intercepted from the incoming plane wave is W = PA where A is the effective aperture of receiving antenna. This power is delivered, perhaps through a transmission line, to some load Y_L as

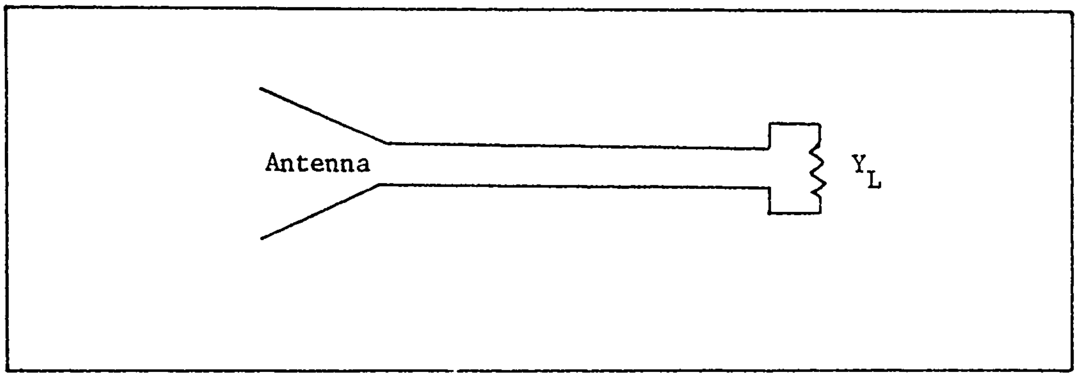

Fig 1a. Antenna to Load Coupling

shown in Figure 1a. Thus a voltage is developed across the load by the antenna whose source susceptance is $Y_a = G_a + JB_a$ as shown in Figure 2.

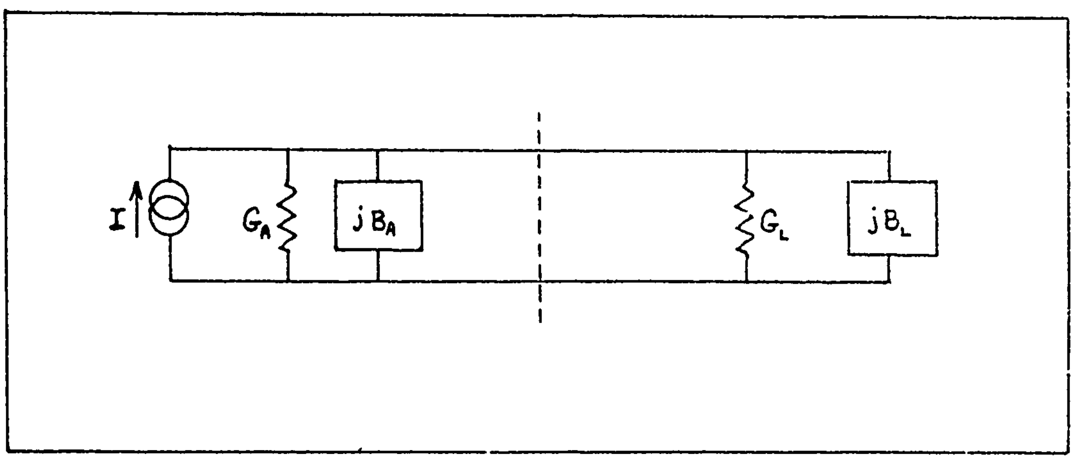

Fig 2. Equivalent Circuit of Fig 1

G_a is the antenna conductance which is composed of G_r (the radiation conductance) and G_{loss} (the loss conductance).

By setting the load power $W = V^2G_L$ the magnitude of the voltage (rms) across the load is found as

$$V_L = \frac{|I|}{\sqrt{(G_a + C_L)^2 + (B_A + B_L)^2}}$$

$$= \frac{|I|}{\sqrt{(G_r + G_{loss} + G_L)^2 + (B_A + B_L)^2}}$$

Relating the load power W to the antenna current and antennas and load admittance yields

$$W = \frac{G_L |I|^2}{(G_r + G_{loss} + G_L)^2 + (B_A + B_L)^2}$$

The effective aperture can now be defined as $A_e = \frac{W}{P}$

$$A_e = \frac{|I|^2 G_L}{P\ (G_r + G_{loss} + G_L)^2 + (B_A + B_L)^2}$$

The current I is that induced when the aperture is oriented perpendicular to the direction of wave propagation and when the aperture has the polarization of the incoming wave.

If the polarization is other than "matched" as above, the effective receiving cross section is reduced (Collin & Zucker, 1969, p 106). The factor by which the received power W is reduced is given by

$$\frac{|\overline{h} \cdot \overline{E}_0|^2}{|\overline{h}|^2 |E_0|^2}$$

where $\overline{E}_0$ is the incident field vector and where $\overline{h}$ is a complex vector (with phasor components h_x and h_y) which defines the state of polarization of the antenna. Thus a new effective aperture, which takes into account polarization mismatch, can be defined as

$$A_e = \frac{|I|^2 \, |\bar{h} \cdot \bar{E}_0|^2 \, G_L}{|\bar{h}|^2 \, |\bar{E}_0|^2 \, P \, [(G_r + G_{loss} + G_L)^2 + (B_A + B_L)^2]}$$

If the load is mismatched, the effective aperture will require another reduction factor $(1-|\Gamma_L|^2)$ (Collin & Zucker 1969, p 106) where Γ_L is the complex reflection coefficient of the load. The resulting effective aperture is

$$A_e = \frac{|I|^2 \, |\bar{h} \cdot \bar{E}_0|^2 (1-|\Gamma_L|^2) \, G_L}{|\bar{h}|^2 \, |\bar{E}_0|^2 \, P \, [(G_r + G_{loss} + G_L)^2 + (B_A + B_L)^2]}$$

Thus it is shown how polarization mismatch simply acts as one of the loss factors in the basic definition of the antenna (aperture).

Speed of Response

Background. In any responsive polarization system that uses microwave components there is some inherent time delay associated with each subsystem building block. Undesirable system delay, when responding to a radar pulse, creates the effect of uncovering the leading edge of the echo pulse; that is, the radar receiver has the opportunity to derive tracking information from the initial part of the echo and the jamming pulse creates tracking error over only the remainder of the pulse time. In a typical situation the radar would send a 500 nanosecond pulse and the responsive jammer would respond 150 nanoseconds later (Van Brunt 1978). The effectiveness of the jammer would be significantly reduced, assuming that the radar can make use of the unjammed portion of the pulse (as in leading edge tracking or operator observing the radar scope).

In the system of interest in this thesis the time delays will accumulate due to (a) transmission paths, (b) detectors, (c) the processor, and (d) active amplifiers. These time delays will be discussed in that order.

(a) Transmission Path. The transmission path delays can be estimated using the rule of thumb that signals propagate in free space at about 1 nanosecond per linear foot. In rectangular waveguide the time will be slightly greater because waves travel obliquely down the guide. Their speed depends on the operating frequency versus the guide dimensions, so the 1 nanosecond value should be multiplied by (Brown, Sharp and Hughes 1961)

$$\frac{1}{\sqrt{1 - (\frac{f_c}{f})^2}}$$

Over the useful range of the waveguide the time delay increase factor is normally less than 3, so a conservative estimate is 3 nanoseconds per foot. Using 4 feet for the path length of the experimental setup, we would expect less than 12 nanoseconds delay. An actual system could be built in miniature microwave components so that the longest path is less than one foot; therefore 3 nanoseconds is a good estimate for the transmission delay of a realizable system. This estimate, of course, omits any connecting transmission lines which might be necessary due to separation of antennas and the system, as on board an aircraft. If such lines are needed, their delay can be estimated at between 1.4 and 3 nanoseconds per foot, depending on whether they are coaxial or waveguide transmission lines.

(b) Detectors. The detectors responding to the microwave signal power will require finite integration time. Assuming square law detection, the detector output is (Papoulis 1965):

$$\int_0^T s^2(t)dt$$

The speed of response is therefore limited by the necessary integration period Tee. In theory, only one half of one microwave signal period is required to produce a detected output. This would require only 0.1 nanosecond for a microwave frequency of 5 giga-Hertz. This is not too practical because the video bandwidth required for such a narrow pulse would itself extend into the microwave region. A more practical value is estimated to be 10 nanoseconds, which allows integration of fifty periods (at 5 gHz) to be integrated before useful output is produced. In addition to reducing the video bandwidth requirements, the longer integration time enhances the signal to noise ratio because the energy is one hundred times higher.

(c) Processor. The processor receives analog signals and performs matrix multiplication on them, followed by arithmetic operations to create the Jones elements. This is followed by a matrix multiplication to create the output amplitude and phase modulating signals for the transmitter. Since the processor was not built, only a rough estimate of time requirement can be made (Kuck, Lawrie and Samek 1977).

If digital processing is employed, then analog-to-digital conversion and back is expected to require about 15 nanoseconds for an 8 bit device. The arithmetic operations are estimated to require an additional 50 nanoseconds for a total of 65 nanoseconds (Tomovit and Karplus 1962). This assumes that a special purpose processor is designed for the system.

It is conceivable that the processor could be entirely analog, which should reduce the time an order of magnitude.

(d) Active Amplifiers. If active amplifiers are used, they will add delay time due to the system. Traveling wave tube amplifiers are commonly used in the microwave frequency region to deliver several kilowatts of peak power. Of course, phase tracking would be necessary with a TWT in the horizontal transmitter channel and another TWT in the vertical channel. The delay time through a highpower TWT is estimated to be 20 nanoseconds (Van Brunt 1978).

Total Response Time

The entire system should, from these estimates, be capable of response within (3+10+65+20=98) less than 100 nanoseconds, well within a typical radar pulse width. Interconnecting transmission lines will increase the delay as discussed earlier.

III System Concepts

Having identified the theoretical tools we now turn to using these mathematical methods in a system sense to create a flexible responsive polarization device. The device should unambiguously recognize the exact polarization of any incoming plane electromagnetic wave and respond with a new wave having polarization characteristics different from those of the incoming wave by any desired degree, as programmed by external control signals. The control signals could originate from either a human operator or a predetermined polarization modulation program.

Responsive polarization refers to processing an incoming radio frequency (or microwave) electromagnetic wave so that it's polarization characteristics can be changed prior to retransmission. The new wave is typically transmitted in the direction of the source of the incoming wave.

Responsive polarization requires the interception of the incoming wave, alteration of its polarization characteristics and retransmission of the modified wave. In addition to polarization changes the amplitude of the wave would typically be increased by such a system.

There are many ways to physically modify polarization such as twist reflectors, birefringent crystals, gratings, etc. Flexible control of the polarization response (in sub microsecond time intervals) is a goal of this study, which eliminates physical or mechanical polarization changing devices. We have instead concentrated on electronic polarization concepts.

Four methods to implement a polarization responsive system are outlined here. They are:

(a) detection and averaging of the power in each of four Stokes vector wave components (Kraus 1966). Each Stokes element representation is then modified appropriately, (multiplied by Mueller matrices in a processor) then used to control the modulation of the power of similar components of a reconstituted wave having the desired responsive polarization characteristics. The block diagram of such an inverse Stokes transmitter is given in Fig 3.

(b) The incoming wave could be sensed and represented by a Jones vector, the two elements of which contain magnitude and phase information for two orthogonal components of the wave. The components are sensed by two orthogonal antennas typically linear horizontally and linear vertically polarized. After suitable modification of their relative amplitudes and phases, the two altered RF signals represent the elements of a new Jones vector which is retransmitted (after any desired amplification).

(c) The incoming wave is sensed by two orthogonal antennas as in (b) and the signals of each antenna are coherently up converted to optical frequencies, which results in an optical Jones vector representation. Alteration of that vector is accomplished by optical methods such as sheet polarizers, birefringent polarizers, retarders and lenses. Optical methods are easily described by matrix methods (Gerrard and Burch, 1975).

The modified light waves are intercepted and down converted, coherently, to the incoming RF frequency. The orthogonal components are then retransmitted as in (b). Phase information would need to be preserved in the conversion processes. Chapter 8 of "Introduction to Optical Electronics" (Yariv 1971) describes methods to up convert to optical frequencies. The conversion efficiency is expected to be quite low so the feasibility would require careful investigation.

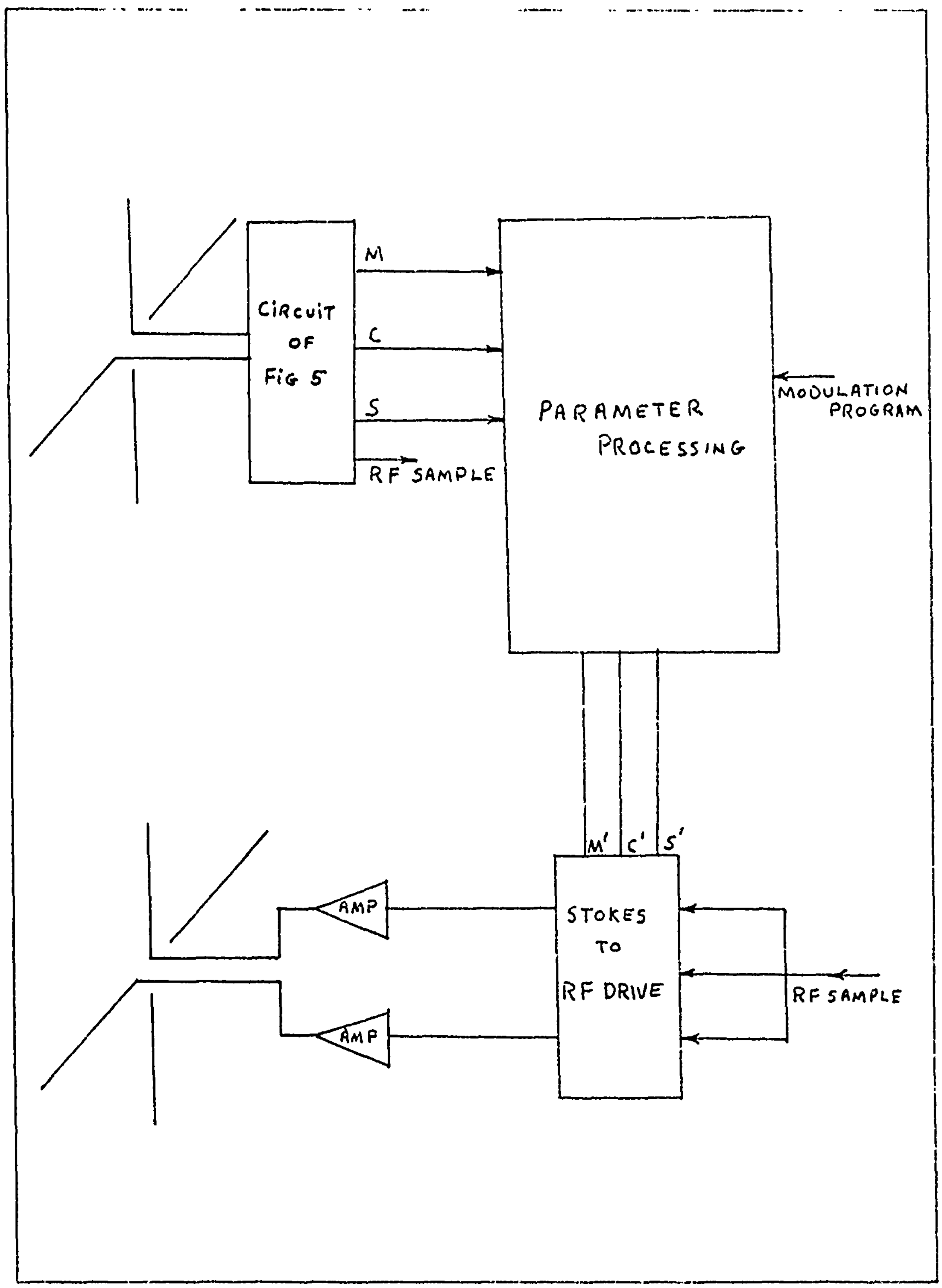

Fig 3. Stokes Receiver/Stokes Transmitter Concept

Also, changing polarization by mechanical methods would be too slow to meet the sub microsecond response goal.

(d) The incoming wave is down converted to an intermediate frequency (IF) (Saxton 1964). The IF signals are altered by amplitude modulators and phase shifters to create the desired elements of the new Jones vector. As in the previous method (c) phase information will require preservation.

These four concepts are included to show some of the alternatives which were considered prior to choosing a system for further study and experimentation.

Method (a) has the advantage that RF phase is neither measured directly nor modulated directly in the transmitter. It does, however, require duplication of much of the receiver hardware (fed in reverse direction) in the transmitter. The circuit complexity is the reason for not choosing method (a) in its entirety.

Method (b) requires RF phase measurement by the receiver; also, the processor would be required to multiply complex numbers. For these reasons method (b) was considered likely to introduce errors and ambiguities in the response.

Method (c) seems least likely to succeed for several reasons. First, the RF to optical conversion efficiency is low, perhaps leading to unnecessarily poor signal-to-noise ratios. The mechanical methods would be too slow to meet the previously discussed speed of response goal. Third, the optical device would require very stable alignment which might be difficult in the shock and vibration environment of an airborne platform (Yariv 1971).

Method (d) offers the selectivity and signal-to-noise advantage of a superhetrodyne receiver. Its disadvantages are lower probability of intercepting the incoming signal (when wideband coverage is required) and the longer time required to integrate a given number of wave periods (lower information rate).

After considering the above, a fifth conceptual method evolved. Its description follows. The receiver portion of concept (a) was chosen for the feasibility experiment of this thesis. The transmitter was chosen as the Jones transmitter of concept (b). A Photograph of such an arrangement is given in Fig 4. This combination was considered the most promising from the implementation standpoint because direct RF phase measurements are not required. Direct RF phase does not enter the process until the output channels are modulated. The selected system concept will next be addressed in greater detail including some implementation ideas.

A Stokes parameter polarization characterization is the mathematical basis of the concept. Each of these parameters is time averaged and therefore involves no direct RF phase information. The result is simple hardware when contrasted to other matrix representations which rely on measuring accurate phase.

The Stokes representation is useful to represent fully polarized waves, unpolarized waves, or combinations of the two. In its most general form the responsive polarization concept described here senses the ratio of polarized to unpolarized wave and responds in a like ratio. The most interesting application of the concept (against polarized radar) involves completely polarized signals; therefore the intensity parameter (I) can be taken equal 1 in the normalized vector with this assumption. The incoming polarized wave can be fully characterized by only three parameters: the preference for horizontal polarization (M),

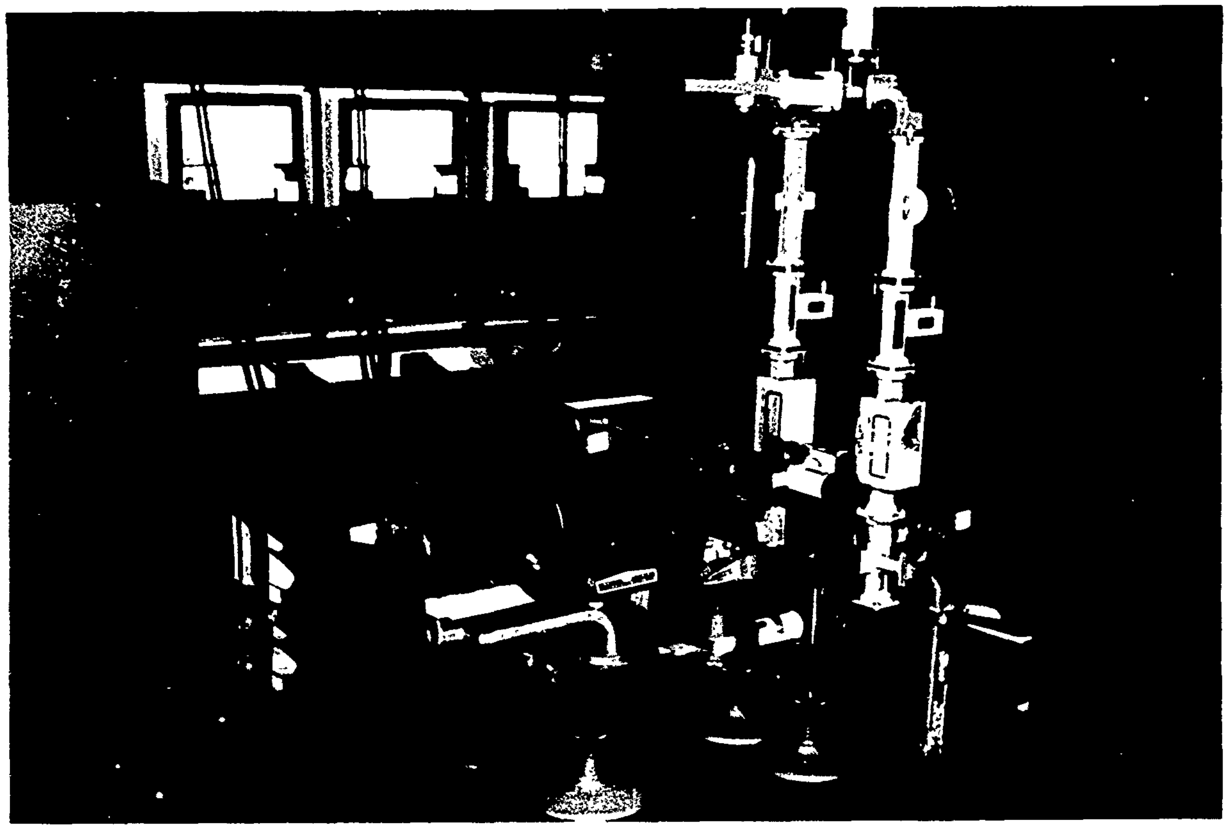

Fig 4. Stokes Receiver with Jones Transmitter

the preferences for slant 45 degrees polarization (C), and the preference for left circular polarization (S) (right circular definition is used by some texts particularly in physics and optics).

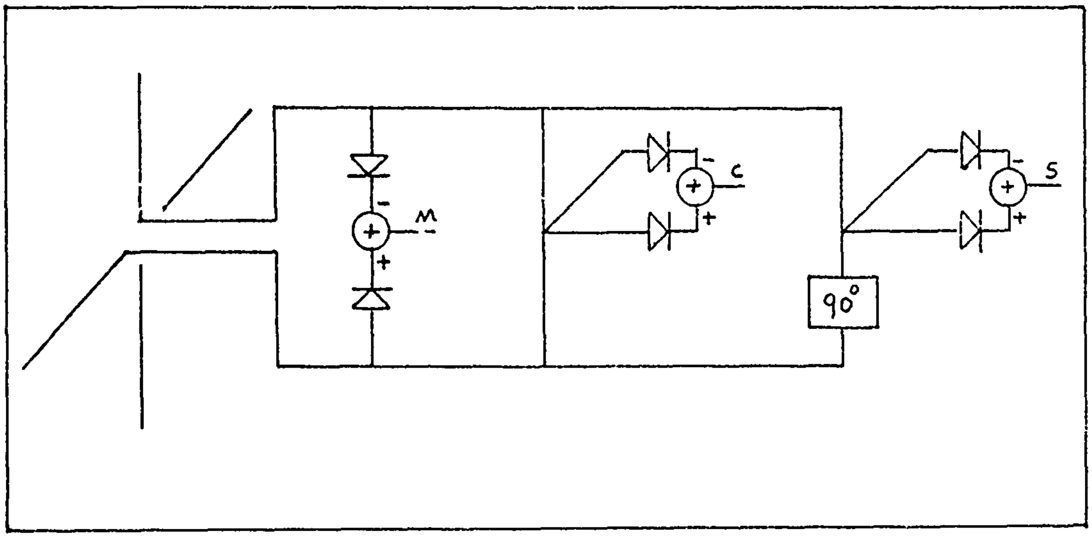

Fig 5. Hybrid Tee Circuit

The RF line length for each of the three parameter paths must be equal and as short as possible. Amplitude normalization might be required, due to the difference in components in each path. The outputs in Fig 5 are signals with amplitudes corresponding to the three Stokes parameters necessary and sufficient to describe the fully polarized wave. Implementing a transmit antenna system with phase-coherent RF drive signals will create a fully polarized responsive wave. Its polarization characteristics will depend on the amplitude and phase of the RF drive signals applied to the transmit antennas. The transmit portion of the selected system also uses a crossed dipole (or horn) antenna pair pointed at the source of incoming signal. Signal processing (including RF amplification) between the sense antennas and the transmit antennas provides the desired polarization control and signal gain (Fig 6). Modulation is applied to the processor to enable programming the desired polarization response in either a steady or time-varying mode.

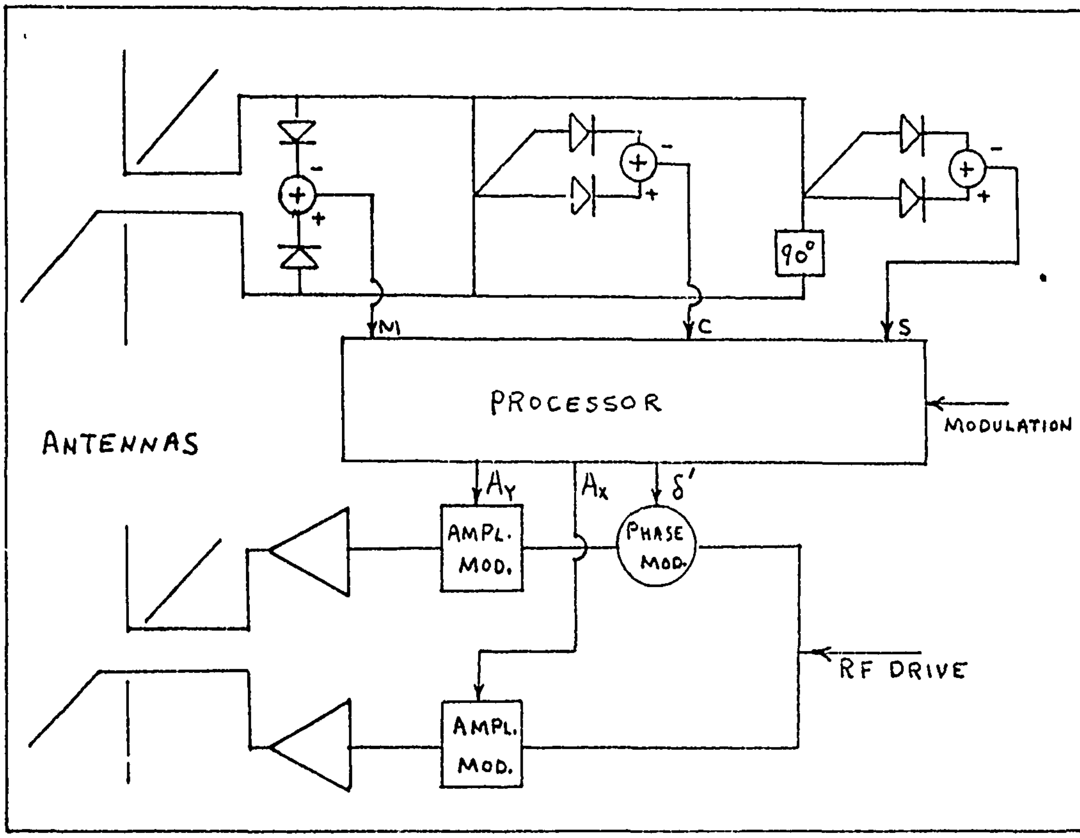

Fig 6. Stokes Receive/Jones Transmit Concept

Using the Stokes parameters (M, C and S) of the incoming signal, the processor creates new parameters M', C' and S', which are used to control the RF drive signals so that the required phase and amplitude Jones vector representation reach the transmit antenna.

Two cases will be used to illustrate the operation:

Example 1. Let the incoming wave be horizontally polarized

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix}$$

We desire an orthogonally polarized (vertical response)

$$\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

The processing would in effect multiply the M C S parameters by -1 (which has the effect of orthogonalizing any incoming polarization).

Example 2. Let the incoming wave be vertically polarized

$$\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

R C P response is desired

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

A negative S' signal would be created by the processor and the M' signal would be driven to zero. Intermediate polarization (right elliptical) would result in fractional primed parameters. In both of the above cases the primed parameters would be converted to the Jones vector form for modulating the transmitter using the computational methods of Appendix E.

IV Matrix Application

A set of linear equations can be express very concisely in matrix notation. By following the rules of matrix algebra, various operations can be performed in a routine, systematic manner. For example, matrix multiplication yields a shorthand expression for multiplying individual elements of two separate matrices, summing certain of these products and forming a new matrix which is the product of the two separate matrices.

Scientists have been using matrix notation for many years to describe optical phenomena. Gerrard and Burch (1975), describe optical matrix methods, including their application to the polarization of light. Kraus (1966) used matrix methods to describe polarization in radioastronomy research. Of course radioastronomy employs much longer wavelengths than optics, but the electromagnetic wave formulation of polarization is the same for optics as it is for radio waves.

In addition to polarization, matrices have been used to handle ray optics, coordinate transformations, propagation and wave reflection/refraction. This discussion will be limited to those linear matrix operations useful for describing the states of a plane monochromatic, electromagnetic wave or the changes introduced by a medium or device as the wave passes through it.

Polarization matrices subdivide into two categories. The first makes provision for including unpolarized energy (also called randomly polarized energy) while the second category deals with fully polarized waves. To handle unpolarized wave components, a 4 x 4 matrix is required. The polarization state is represented by a 4 x 1 vector. Such a matrix is called a Mueller matrix after its inventor. The vector, called a

Stokes vector, was introduced by Sir George Stokes (1852).

The polarization state of a wave passing through a device can be found by the premultiplication of the Stokes vector by the Mueller matrix as follows:

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \times \begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} = \begin{bmatrix} I' \\ M' \\ C' \\ S' \end{bmatrix}$$

If part of the wave is unpolarized, the Stokes vector can be written to the sum of an unpolarized vector plus a fully polarized vector as

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} = \begin{bmatrix} 1-d \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} d \\ dM \\ dC \\ dS \end{bmatrix}$$

where d is the fractional power in the fully polarized part of the wave. For example, a wave that contains 1/3 polarized and 2/3 unpolarized power could be expressed as

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} = \begin{bmatrix} 2/3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/3 \\ M/3 \\ C/3 \\ S/3 \end{bmatrix}$$

It follows that $I = (1-d) + d = 1$ for the example used. That is because normalization of the power in the wave was assumed.

The Stokes elements are time-averaged quantities. Each element represents the average of the instantaneous power in the wave. For example (normalizing the impedance to one)

$$I = \frac{1}{T} \int_0^T \left[E_x^{\,2}(t) + E_y^{\,2}(t) \right] dt$$

$$M = \frac{1}{T} \int_0^T \left[E_x^{\,2}(t) - E_y^{\,2}(t) \right] dt$$

$$C = \frac{1}{T} \int_0^T 2E_x(t)E_y(t)\cos\delta \ dt$$

$$S = \frac{1}{T} \int_0^T 2E_x(t)E_y(t) \sin\delta \ dt$$

Each element of the Stokes vector has an interpretation. The I element is intensity of the wave. M is the preference for horizontal polarization, which can be observed in the above equation. C is preference for slant 45 degree linear polarization and S is preference for left circular polarization.

If the wave is fully polarized, then a simpler 2 x 2 matrix can be used to premultiply a 2 x 1 vector. Unlike the Stokes and Mueller elements, these elements must contain phase information. The simpler calculus was first used by R. Clark Jones (1941) to describe polarization; hence the names Jones matrix and Jones vector apply.

The Jones vector elements are magnitude of field (usually taken as the electric field) with phase term $e^{j\delta}$. A polari..ation modifier operating on an incoming wave could be stated as

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \times \begin{bmatrix} E_x e^{j\delta_x} \\ E_y e^{j\delta_y} \end{bmatrix}$$

Here the phase term e^{jwt} due to radio frequency carrier is suppressed since it is unchanged by the polarization modifier.

Jones representations are often expressed in terms of phase difference between E_x and E_y as $\delta = \delta_y - \delta_x$. That is because absolute phase, referred to some starting phase, does not enter into the polarization description. All that is needed to describe the polarization state is relative phase between the x and y components.

In either the Jones or the Mueller-Stokes calculus chain, matrix multiplications can be used to describe the wave passing through multiple devices.

The Jones vector can be expressed in terms of Stokes vector since polarized waves are a subset of the more general set containing randomly polarized energy (see Appendix E). The reverse transformation is in general not possible unless complete polarization is assumed.

The Poincare sphere is a convenient method to visualize the polarization state. Letting $\gamma = \tan^{-1}\frac{|E_y|}{|E_x|}$ where γ is between 0 and 90° and δ is the phase difference as before, then the polarization state is a point on the surface of the sphere located as shown in Figure 7.

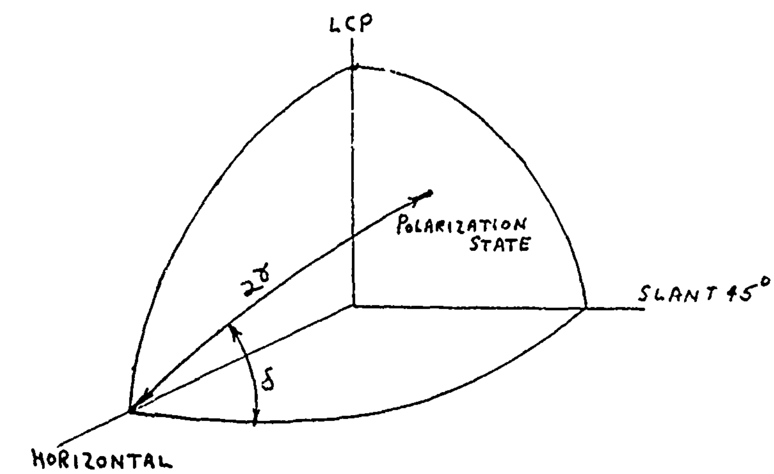

Fig 7. Poincare Sphere Representation of Polarization State

The quantity 2 γ is a great circle angular distance from the horizontally polarized axis and δ is the angle of that great circle line with respect to the equator (linearly polarized great circle). In this representation the top or north pole represents left circular polarization while the south pole represents right circular polarization (IRE definition).

If part of the wave energy is unpolarized, the Poincare sphere shrinks from a radius of 1 to a radius of d. The unpolarized portion can be visualized as all points on the surface of another sphere of diameter = 1-d.

From the above discussion it is seen that the Jones calculus and the Mueller-Stokes calculus can be quite useful for describing polarization changes that occur through media or devices. This is especially true when multiple media (or devices) are encountered by the wave, in which case ordinary algebra, applied to the simultaneous equations, becomes extremely complex. In addition to the polarization transformations, there are other useful matrix equations which take complicated electromagnetic wave calculations and set them forth as a straightforward process. Some examples follow (Cornbleet, 1976):

Reflected Waves

Transmission and reflection from a plane surface can be described in terms of a 4x4 matrix (different from the Mueller matrix) which operates on a 4x1 column vector representing the incident electric field components and the reflected components on the other side of the boundary (Cornbleet, 1976, p 302). This is expressed in matrix notations as

$$\begin{bmatrix} E_{x_2} \\ E_{y_2} \\ E_{x_3} \\ E_{y_3} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & T_{11} & T_{12} \\ R_{21} & R_{22} & T_{21} & T_{22} \\ T_{11} & T_{12} & R_{11} & R_{12} \\ T_{21} & T_{22} & R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} E_{x_1} \\ E_{y_1} \\ E_{x_4} \\ E_{y_4} \end{bmatrix}$$

where the notation is that of Fig 8 below.

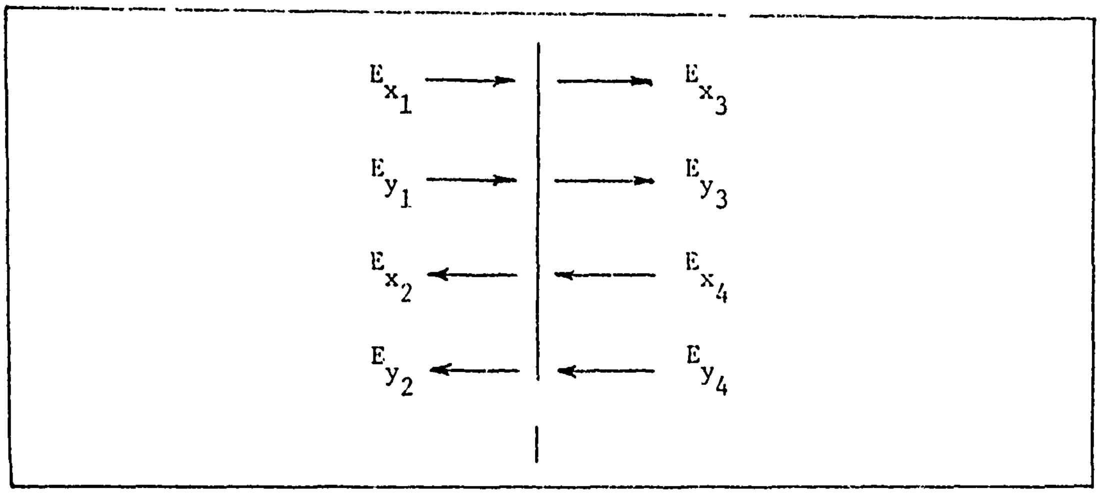

Fig 8. Transmission and Reflection at an Interface

R and T refer to the voltage reflection and transmission coefficients respectively. By this method the power loss due to reflections at an interface can be handled. Cornbleet illustrates how this can be extended to n surfaces very simply, using matrix equations.

<u>Transmission Matrix</u>

A normally incident plane wave impinging on a uniform layer of dielectric material (thickness d) will result in total fields on the incident side given by Cornbleet (1976) as

$$\begin{bmatrix} E_{inc} \\ H_{inc} \end{bmatrix} = \begin{bmatrix} \cos\beta d & \frac{j\omega\mu}{\beta}\sin\beta d \\ \frac{j\beta}{\omega\mu}\sin\beta d & \cos\beta d \end{bmatrix} \times \begin{bmatrix} 1 \\ \frac{\gamma'}{j\omega\mu'} \end{bmatrix} \times E_{ext}$$

where $\beta = \omega\sqrt{\mu\varepsilon}$ is the propagation constant of an unbounded wave in the dielectric. The medium on the transmission side is assumed complex (for generality) with constants ε , μ and propagation constant γ' .

For oblique incidence the matrix equation becomes

$$\begin{bmatrix} E_{inc} \\ H_{inc} \end{bmatrix} = \begin{bmatrix} \cos A & \frac{j}{Y}\sin A \\ jY\sin A & \cos A \end{bmatrix} \begin{bmatrix} 1 \\ Yext \end{bmatrix}$$

where $A = \frac{2\pi d}{\lambda}\left(\varepsilon_r\mu_r - \sin^2\theta\right)^{1/2}$

$$Y_{\perp} = \frac{Y_o\left(\frac{\varepsilon_r}{\mu_r} - \sin^2\theta\right)^{1/2}}{\cos\theta}$$

= media admittance for a perpendicularly polarized plane wave (electric vector perpendicular to the plane of incidence)

$$Y_{11} = \frac{Y_o \frac{\varepsilon_r}{\mu_r}\cos\theta}{\left(\frac{\varepsilon_r}{\mu_r} - \sin^2\theta\right)^{1/2}}$$

= media admittance for a parallel polarized wave (electric vector parallel to the plane of incidence)

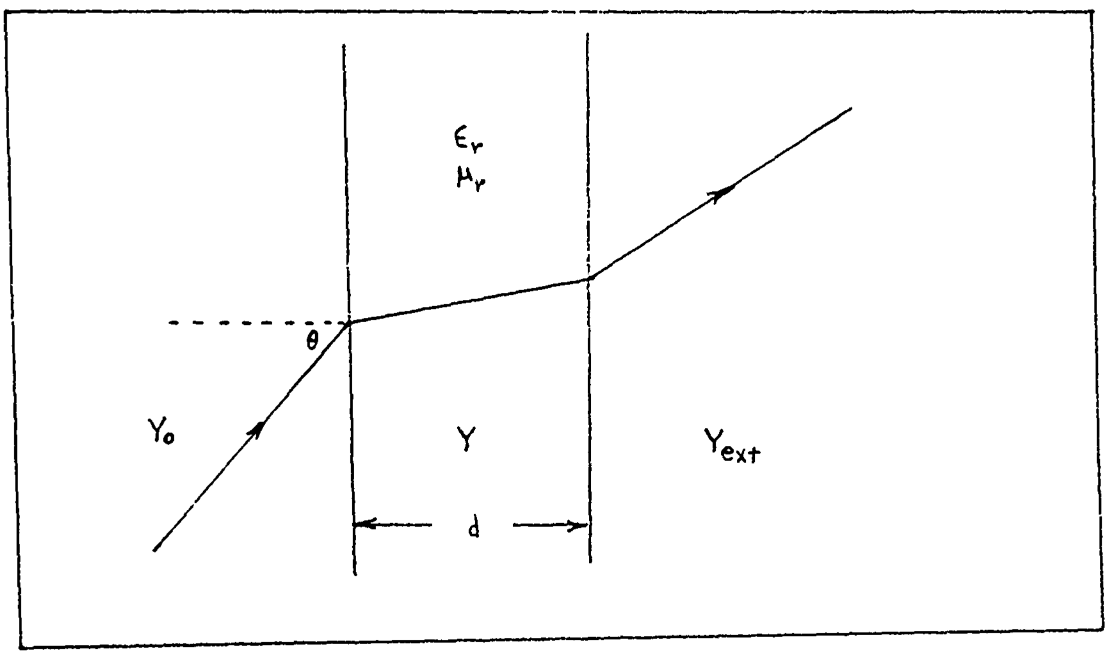

Fig 9 . Oblique Incidence at an Interface

This result was obtained, in matrix form, from the boundary value solutions (continuity of field components of the interfaces). The matrix notation simply provides compact notation. The benefits of the compactness increase when multiple layers of dielectric are involved. Then the matrix equation becomes a chain multiplication of the individual matrices (Cornbleet, 1976, p 162-178).

Polarization Rotators

Two half wave phase shifters, whose fast axes make an angle of 45°, can be used to rotate the direction of wave polarization by 90°. This can be illustrated by assuming an incident plane wave of arbitrary polarization. In Jones vector notation, the wave is

$$\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

where α is the angle between the horizontal axis and the polarization vector. Passing the wave through the two phase shifters (assume their orientation is $\gamma_1 = 22\frac{1}{2}°$ and $\gamma_2 = 67\frac{1}{2}°$), the output polarization is (Cornbleet, 1976, p 311):

$$\begin{bmatrix} \cos 2\gamma_1 & \sin 2\gamma_1 \\ \sin 2\gamma_1 & -\cos 2\gamma_1 \end{bmatrix} \times \begin{bmatrix} \cos 2\gamma_2 & \sin 2\gamma_2 \\ \sin 2\gamma_2 & -\cos 2\gamma_2 \end{bmatrix} \times \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} = \begin{bmatrix} \sin\alpha \\ \cos\alpha \end{bmatrix}$$

which is the input rotated -90°.

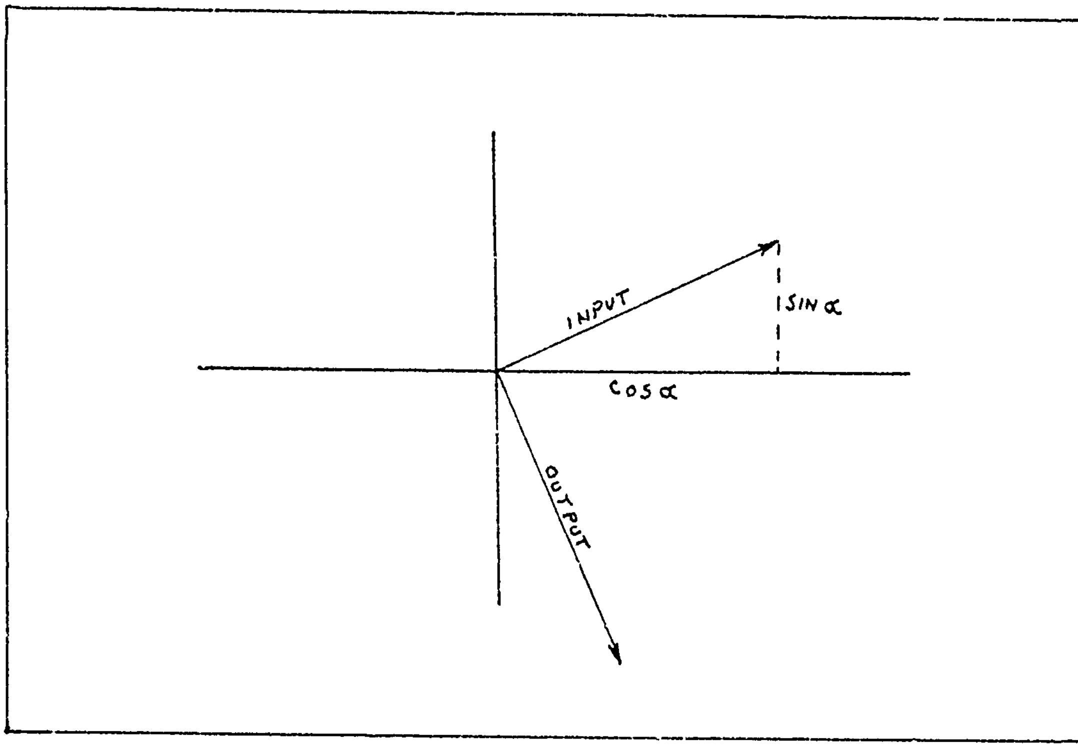

Fig 10. Polarization Rotation

Spin Matrix

The spin matrix is useful in studies of imperfect or irregular conducting surfaces or of the properties of dielectric materials in the form of thin films backed by perfect conductors. This is accomplished through the measurement of the ellipticity of a reflected wave arising from an obliquely illuminating, linearly polarized wave. The procedure could well be adapted to the measurement of the dielectric properties of materials at microwave frequencies; for example such dielectric properties at elevated temperatures, or the thickness of transparent layers.

V Equipment

The equipment for the thesis experiment was chosen to make maximum use of components available in the AFIT microwave laboratory, to be small enough for a bench setup and to avoid any special or unusual hardware.

This section will describe the equipment used to generate data during summer and autumn quarters 1980. The experiment and resulting data are intended to show verification of the theory involved in the system concepts.

The experimental setup models only the baseband Stokes and Jones concepts. The digital processor, instead of being modeled in hardware was functionally simulated by a Fortran Program on the CDC 6600 base computer (See Appendix D).

The baseband Stokes experimental receiver hardware will first be described. This will be followed by the source, antenna mount and the anechoic chamber. Finally the Jones transmitter hardware will be covered.

The experiment uses I band microwave components. Waveguide for I band is 0.9 inches by 0.4 inches inside dimension which propagates the TE_{10} mode at the test frequency. Signal for the experiment was supplied by a Hewlett Packard 620A signal generator amplified by a Hewlett Packard 495A traveling wave tube to about 1 watt (Fig 11).

The Stokes Receiver

The Stokes receive subsystem includes one vertically polarized horn antenna and one horizontally polarized horn antenna to orthogonally intercept the incoming wave components. The antennas can be seen in Figure 4. Each antenna connects to a 20 dB (nominal) directional

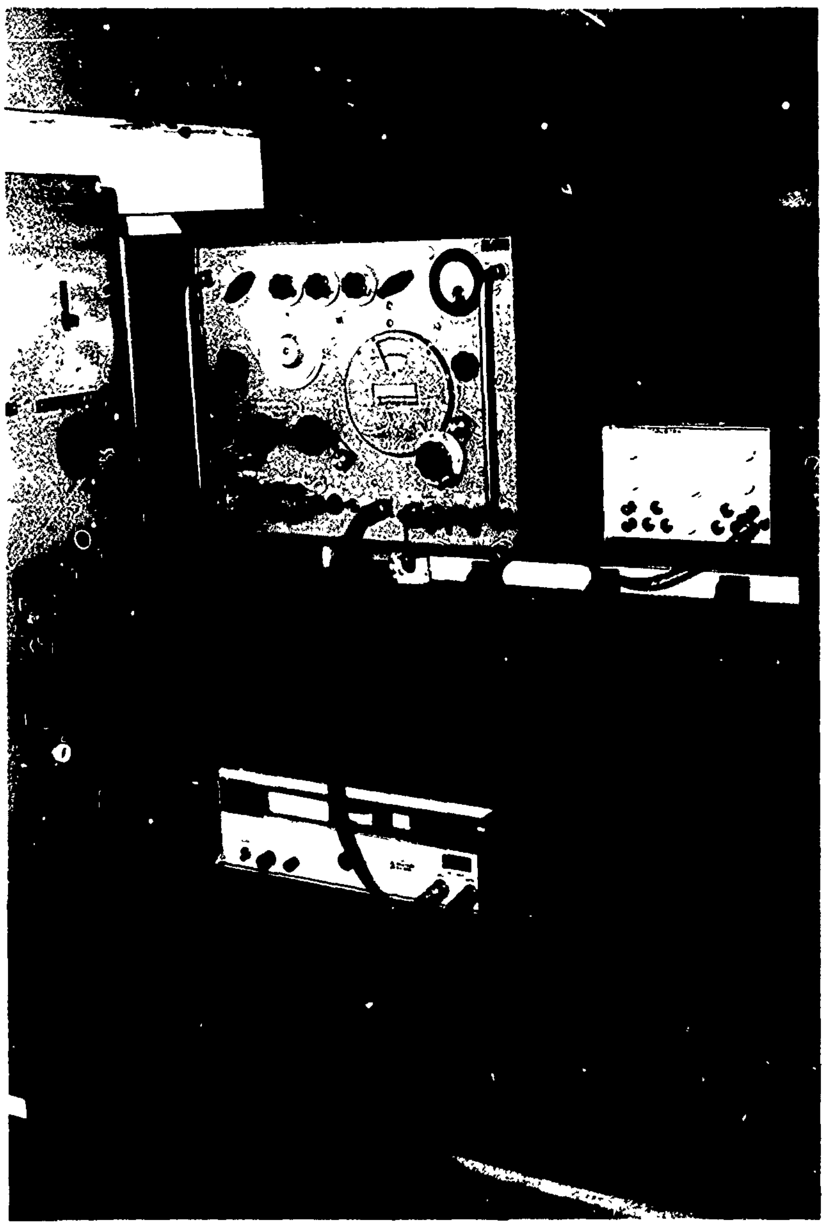

Fig 11. The Signal Source

coupler whose directional port terminates in a tunable waveguide detector mount. A six inch 90° twist section of I band wave is used in the vertical channel to physically align the vertical and horizontal channel waveguides. This was necessary to get the physical symmetry required to fit the rest of the subsystem together. To assure electrical length equivalence through the 90° twist and the straight section in the horizontal channel, the two components were compared using a waveguide slotted line to detect standing wave minimum positions when a brass shorting plate was placed at the end of each component under test. The result was that the twist and straight sections are so identical in electrical length that the smallest available waveguide shim could not be used to improve the match between their lengths.

Following the twist and straight sections, after some necessary waveguide bends, each channel (vertical and horizontal) feeds the sum port of a hybrid (magic) Tee which functions as a power divider. The difference arm of each hybrid Tee is terminated in a matched waveguide load. From the power divider half the power in each channel flows to the input arm of another hybrid Tee where the vertical signal is summed with the horizontal to get slant 45° response out of the sum arm of the Tee or -45° response out of the difference arm. Each arm feeds a tunable waveguide detector mount. It was necessary to use ferrite isolators to isolate the detector mounts. This avoids unbalancing the hybrid Tee due to load mismatch.

A similar arrangement creates circularly polarized response from the other half of each power split signal. The difference is that a variable phase shifter, adjusted to 90° phase shift, is inserted in the vertical channel. When the horizontal signal is summed (in another hybrid Tee) with the phase shifted vertical signal, right circular response is obtained. The difference port of the hybrid Tee yields left circular response. Here ferrite isolators were also necessary to maintain balance in the hybrid Tee.

Ferrite isolators were also placed in the vertical and horizontal channels feeding the circular polarization hybrid to avoid interaction between that hybrid and the 45° polarization hybrid.

Bolometers were used to detect the signals in each of the six tunable waveguide detector mounts. They were chosen over crystal detectors for their true RMS response and their ability to maintain accurate readings over wide dynamic ranges (tracking accuracy).

Each bolometer was fed to a standing wave indicator whose decibel scale provided readings relative to a zero dB reference level common to all six detected signals. Figures 12, 13, 14, 15 and 16 show the components identified above.

The Test Antenna Mount

A heavy metal bunsen burner stand was used to support the transmit test antennas. A coaxial rotary joint provided rotation of each test antenna about its longitudinal axis. Different test antennas were installed by disconnecting the type N connector from the rotary joint. The test antenna mount can be seen in Fig 17.

The Anechoic Chamber

The anechoic chamber is an absorber lined plywood box using no metal fasteners in its construction. It is four feet long by 2 feet 8 1/2 inches square. One end is closed except for a 2 inch diameter hole which permits the transmit antenna to be inserted. The absorber for the side walls is Eccosorb CV-4W while the closed end is covered

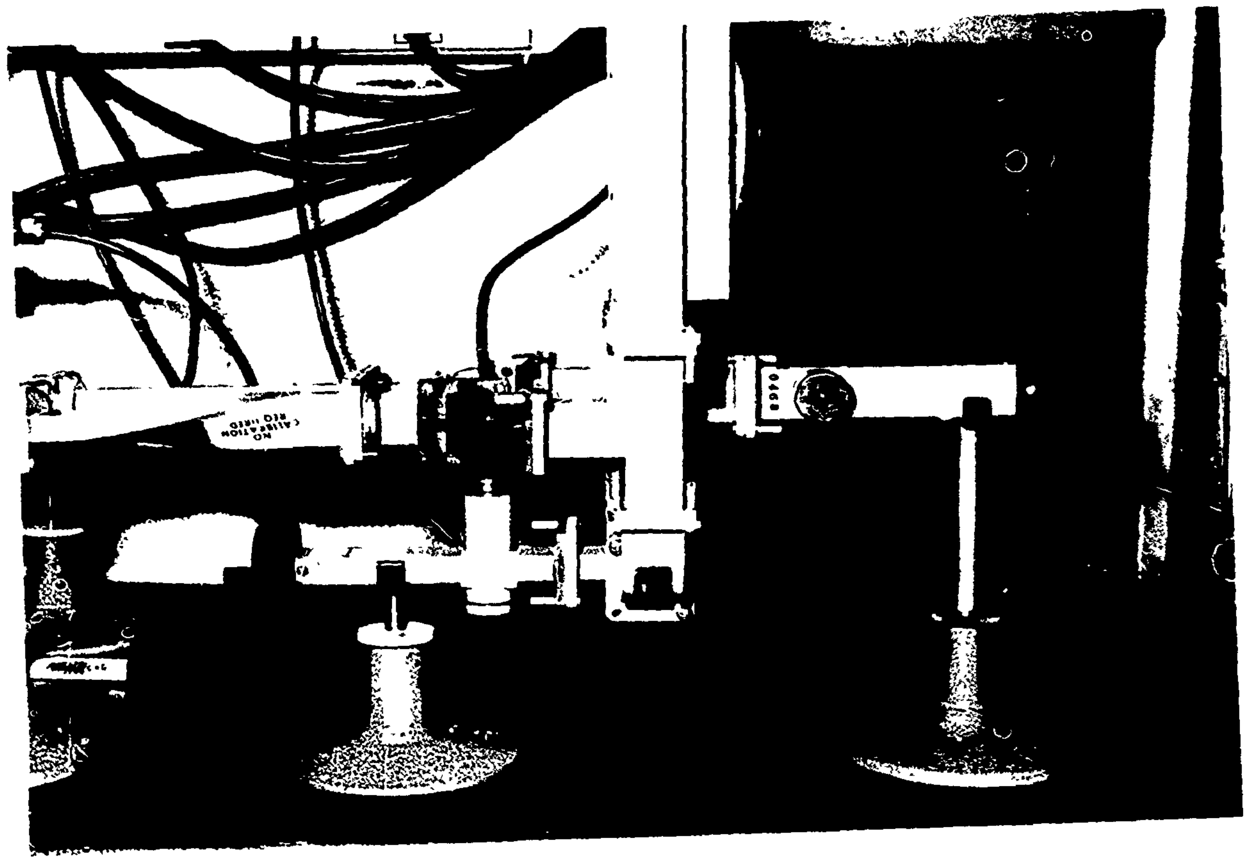

Fig 12. View Showing Detector Mounts

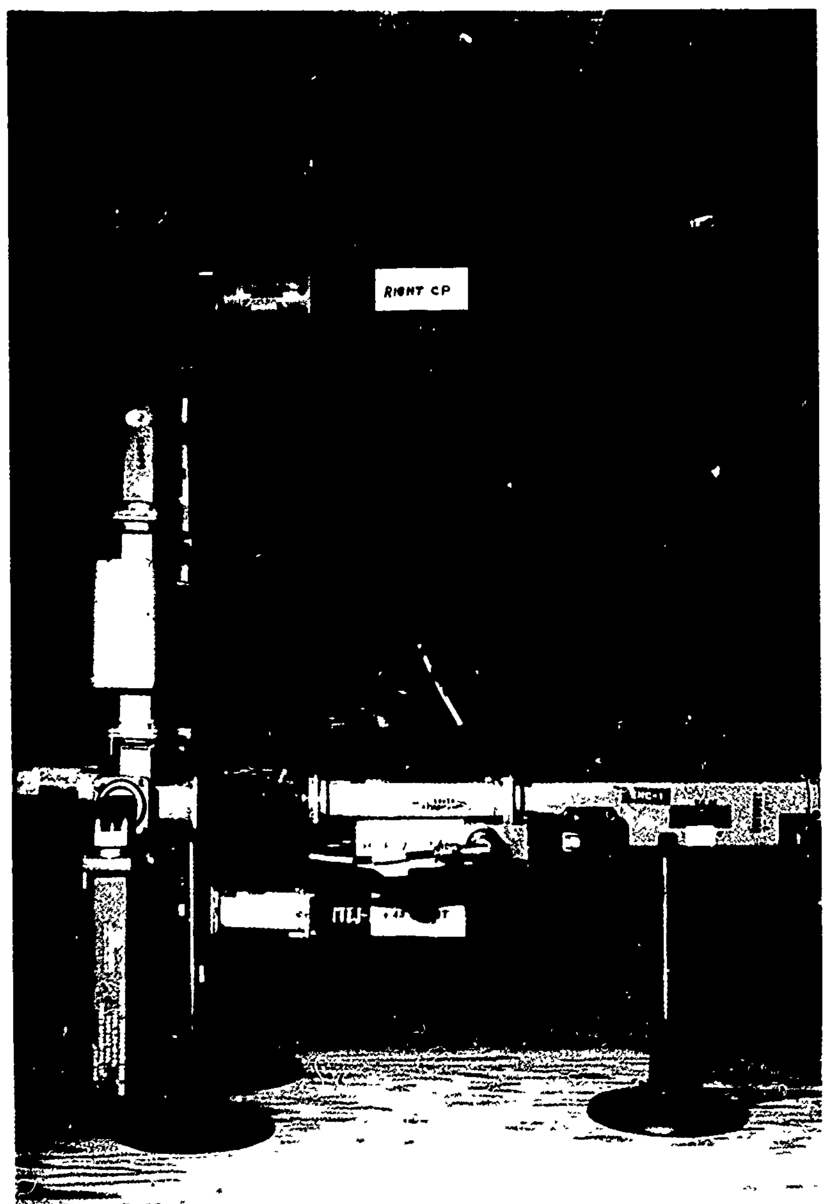

Fig 13. View of Microwave Components

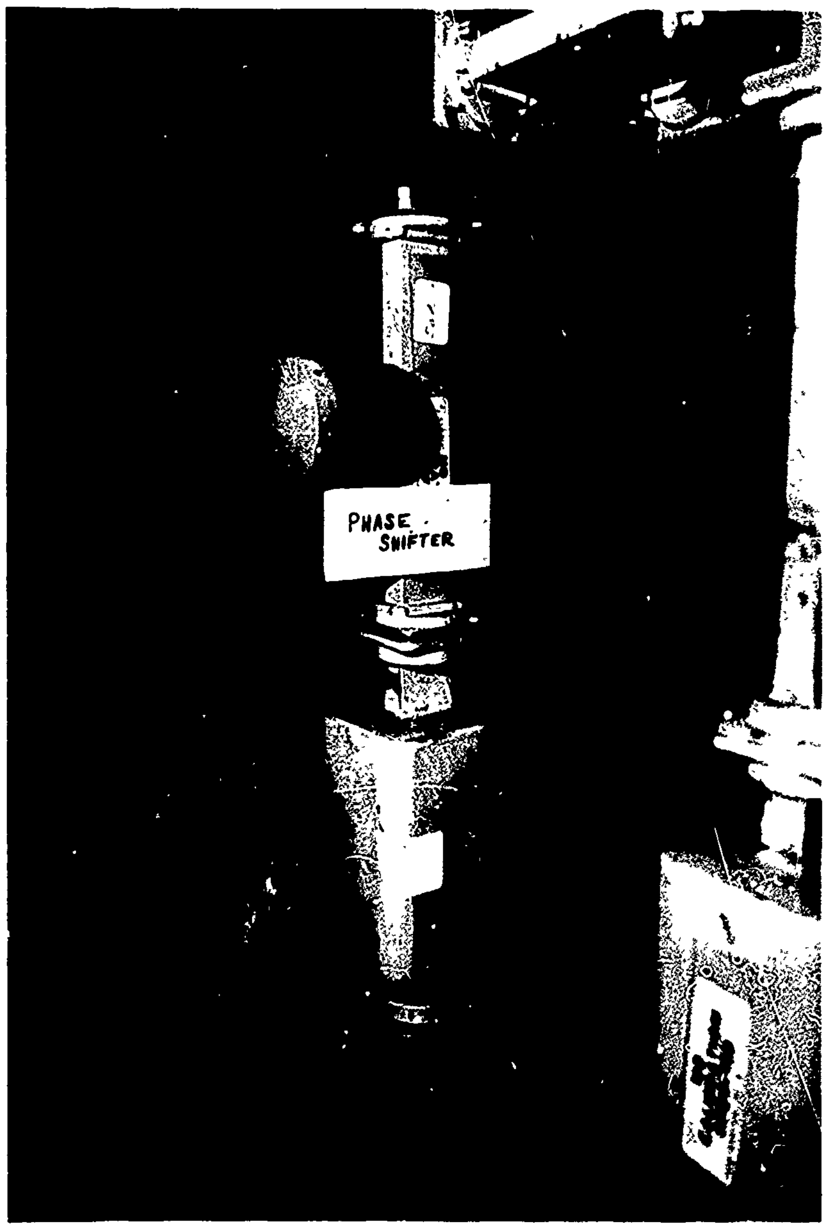

Fig 14. The 90° Phase Shifter

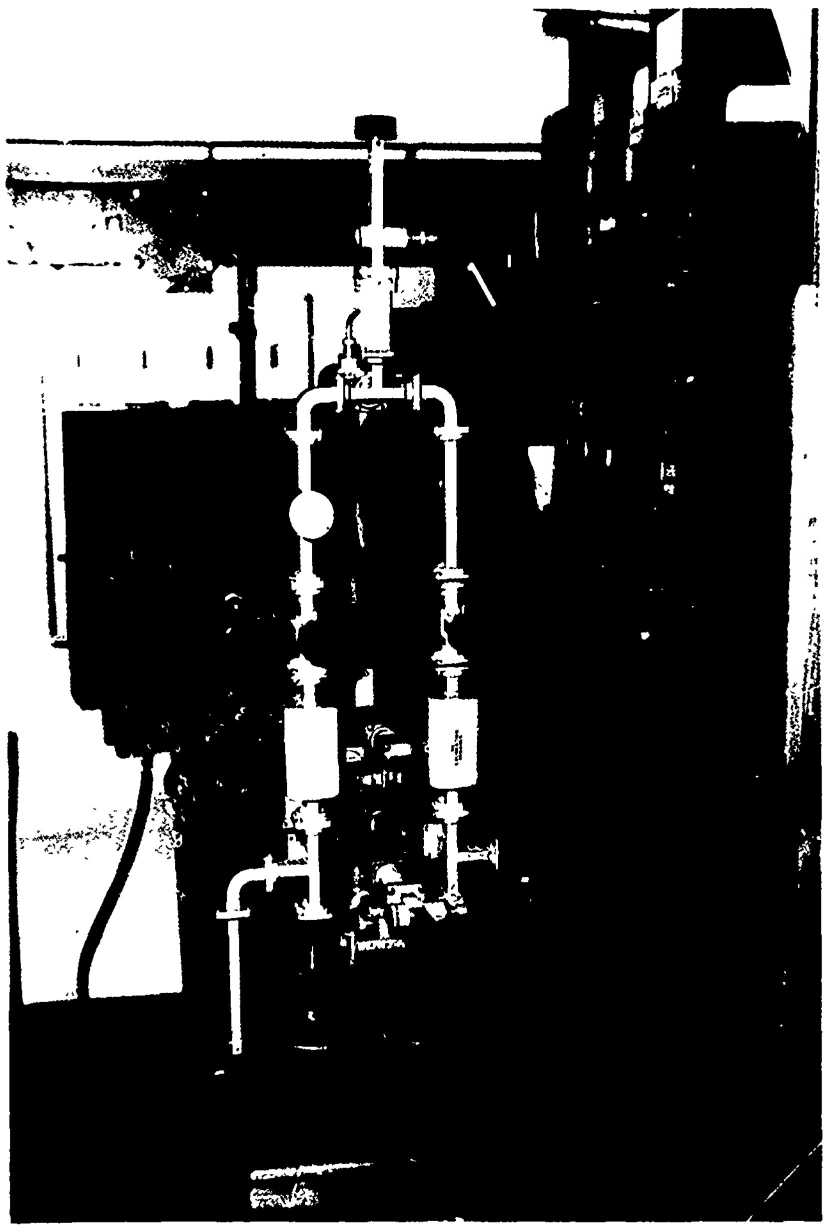

Fig 15. The Experimental Hardware

Fig 16. A View Looking into the Anechoic Chamber

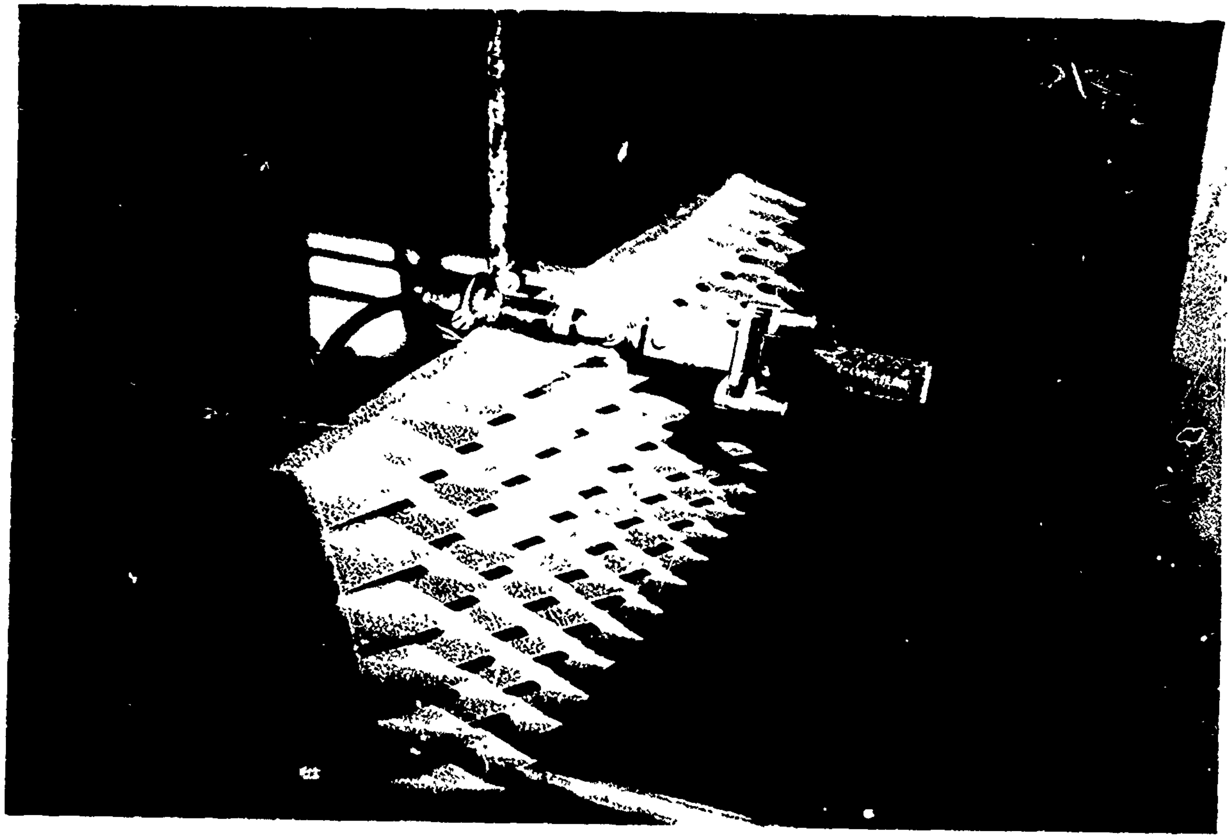

Fig 17. Rotary Joint with the LCP Antenna Installed

with AN-73. Both absorber types permit less than 1 percent reflected energy at normal incidence. The manufacturer claims that performance is relatively insensitive to incidence angle, but quotes no numerical values in the technical bulletin. The box is made of 1/2 inch plywood which allows two person portability. The open end accepts the receiving apparatus. The chamber can be seen in Figures 18 and 19.

The Jones Transmitter

The Jones transmitter uses 1 band components to create orthogonal (vertical and horizontal) signals. It was built to accept a type N coaxial input which is power split, half to the vertical channel and half to the horizontal channel. Each channel has an adjustable attenuator. A calibrated adjustable phase shifter is included in the vertical channel. Each channel feeds a transmit horn antenna identical to those used for the Stokes receiver experiment.

The Jones transmitter was built to fit on the Stokes receiver so that the four horn apertures are in the same plane. This configuration represents the complete responsive polarization system. All that is needed is a digital or analog processor to operate on the receive analog signals and provide amplitude and phase settings for the Jones transmitter.

Equipment Changes

As seen in the various photographs, improvements were made from time to time. For example hybrid Tees, with the difference arm terminated in a matched load were substituted for the plain Tee power dividers. Also ferrite isolators were added as required to avoid interaction between the components due to reflected energy.

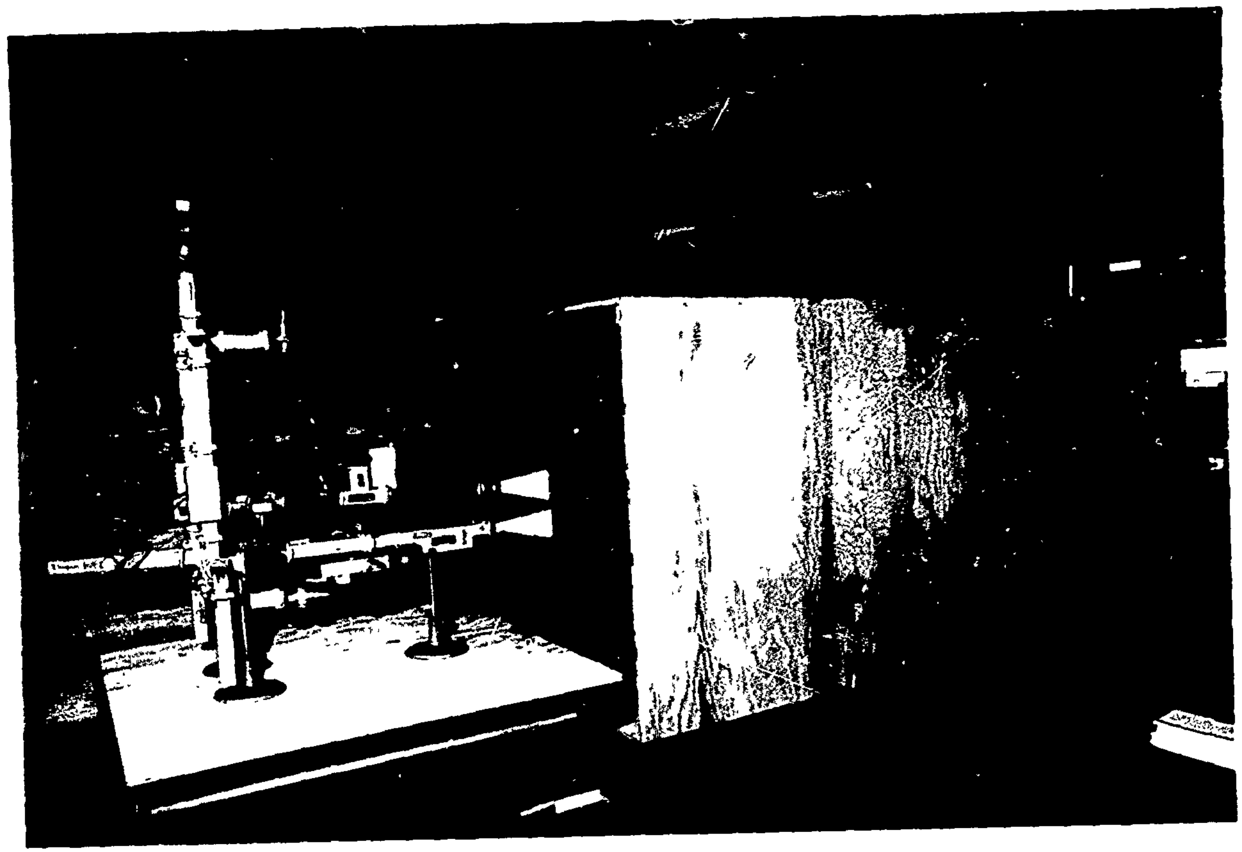

Fig 18. Anechoic Chamber (Oblique View)

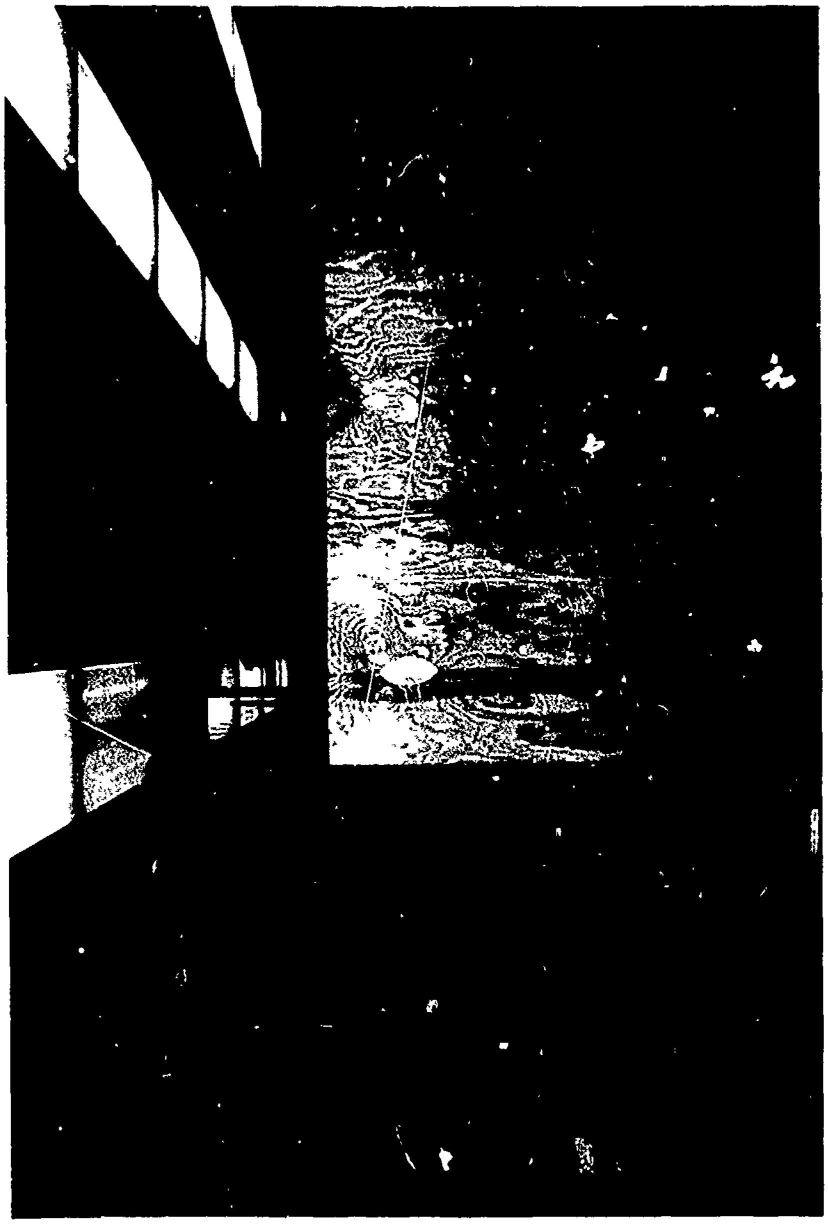

Fig 19. Anechoic Chamber (Side View)

VI Procedure

The experimental part of the thesis involves a feasibility demonstration of the polarization measurement and transmitting concepts. The following procedures evolved from that experimental effort. Stokes parameters were chosen to implement the polarization receiver portion of the conceptual system while a Jones representation was selected to implement the transmitter concept.

Measurements were taken of several test antenna configurations as shown in Fig 20. The antennas include linearly polarized, right circular, left circular, and left circular with a high axial ratio.

The procedure used in the measurement is first described followed by the mathematical procedure for reducing the resulting data. An example measurement, its data and reduction of that data concludes this section.

Calibration

The data measurement procedure begins with a 15 minute warmup to assure that the RF and modulator frequencies have reached their most stable state and that the power output from the signal generator and traveling wave tube amplifier has stabalized.

The output test frequency is next adjusted to exactly 9.34 GHz. The choice of specific test frequency resulted from earlier measurements in which a quarter wave section of waveguide was used in the setup instead of the adjustable phase shifter. 9.34 GHz was the exact frequency at which the available waveguide section measured one quarter guide wavelength. The quarter wave section is no longer needed because a variable phase shifter was found and put into the setup.

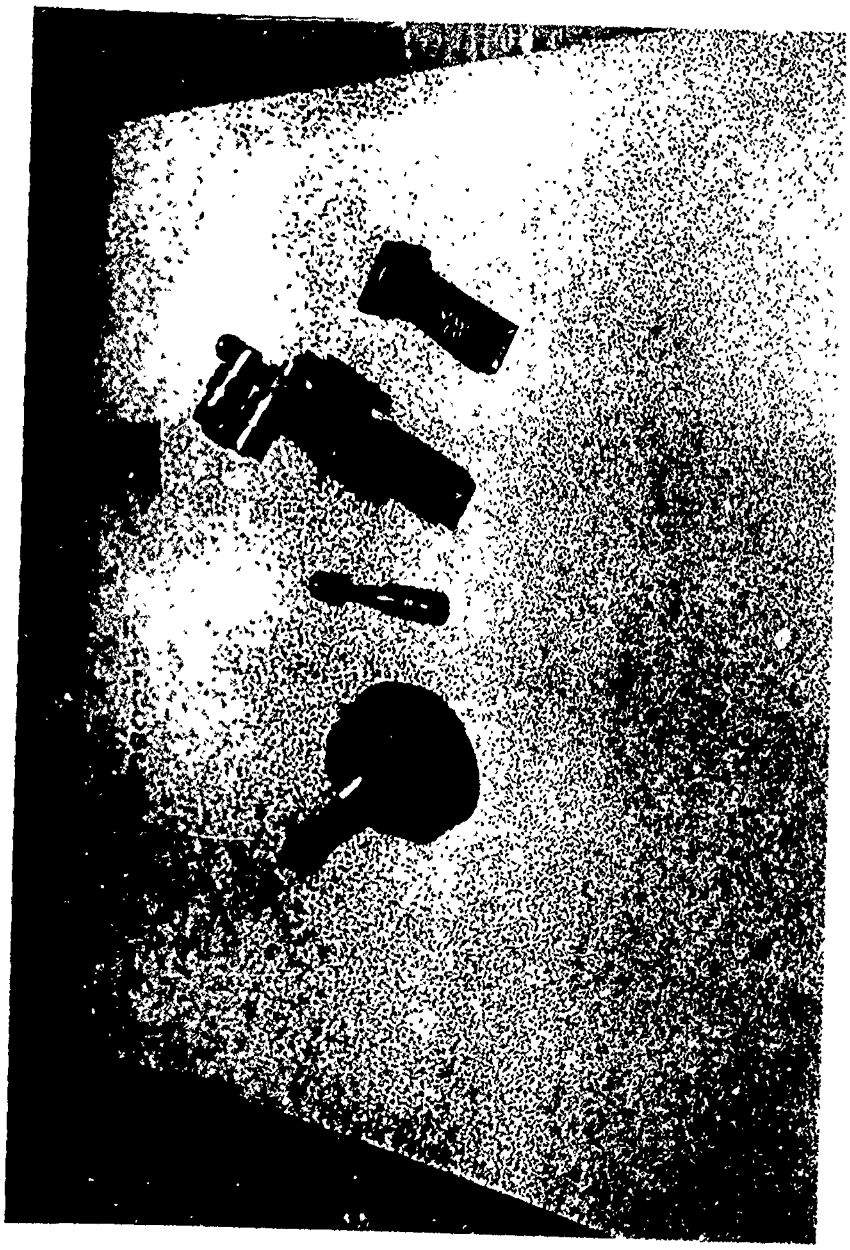

Fig 20. Four Test Antennas

The Stokes receiver (Fig 15) is calibrated by first tuning each detector mount for maximum output on its respective indicator (HP415). The signal level should be approximately that used for the measurement.

The variable phase shifter is adjusted by illuminating both receive antennas with a linearly polarized horn excitation antenna mounted so that it can rotate about its longitudinal axis. A coaxial rotary joint permits the required axial rotation. The separation between the excitation antenna and the receive antenna apertures should be at least 2 feet.

The phase should be initially set for approximately 90° and the final adjustment made by rotating the excitation antenna on axis while observing the right circular and left circular power indicators. Minimum meter movement indicates that the phase shifter is set to precisely 90°. The "minimum movement" should occur on both right and left circular indicators for the same phase shifter setting.

Next a separate linear horn antenna is used to feed the microwave signal from the traveling wave tube (TWT) antenna into each receiver antenna (vertically polarized and horizontally polarized). A flexible coaxial cable between the TWT and the separate antenna permits orienting its aperture directly over the aperture of each receiver antenna. To avoid metal to metal contact, which disturbs the readings, a thin piece of duct tape is used to cover the outer edges of the mouth of the excitation antenna. For this calibration the attenuator of the signal generator is set at about -30 dBm.

With the excitation antenna centered over the vertical receiver antenna the vertical power indicator (HP-415) is adjusted for full scale or 0 dB. Also the +45° indicator and the -45° indicator, the right circular indicator and the left circular indicator are each adjusted to read -3 dB on their respective meter scales.

Next the excitation horn is positioned over the horizontally polarized receive antenna. The horizontal indicator is then adjusted to read 0 dB. As a check, the 45° indicator and the -45° indicator should again read -3 dB as should the right circular indicator and left circular indicators.

The Jones transmitter (Figs 21 and 22) is calibrated by feeding a signal from the signal generator and TWT amplifier into its type N to waveguide adapter. A flexible coaxial cable carries the signal from the TWT to the Jones transmitter. A linear horn sense antenna with crystal detector is connected to a power indicator (HP 415). By placing the sense antenna over each Jones transmit antenna (vertical and horizontal) separately the Jones variable attenuators can be adjusted for the required amplitude ratio. The initial or zero phase setting of the Jones phase shifter is obtained by mounting the sense antenna about 2 feet in front of the Jones transmitter on a coaxial rotary joint. The polarization of the sense antenna is physically set to 45° then the Jones phase shifter is adjusted for maximum indicator reading. A more precise setting can be obtained by adjusting the Jones phase shifter 3 dB below maximum on one side of maximum, noting the phase shifter dial reading, then adjusting it to 3 dB below maximum on the other side of maximum. The mean of the two readings gives the zero phase position. Required phase difference between the vertical and horizontal channels of the Jones transmitter can be added to the zero phase reading to obtain the required setting of the Jones phase shifter.

To measure data the unknown test antenna is mounted on the coaxial rotary joint and pointed through the 2 inch diameter opening in the end

Fig 21. Jones Transmitter Showing Front of Phase Shifter

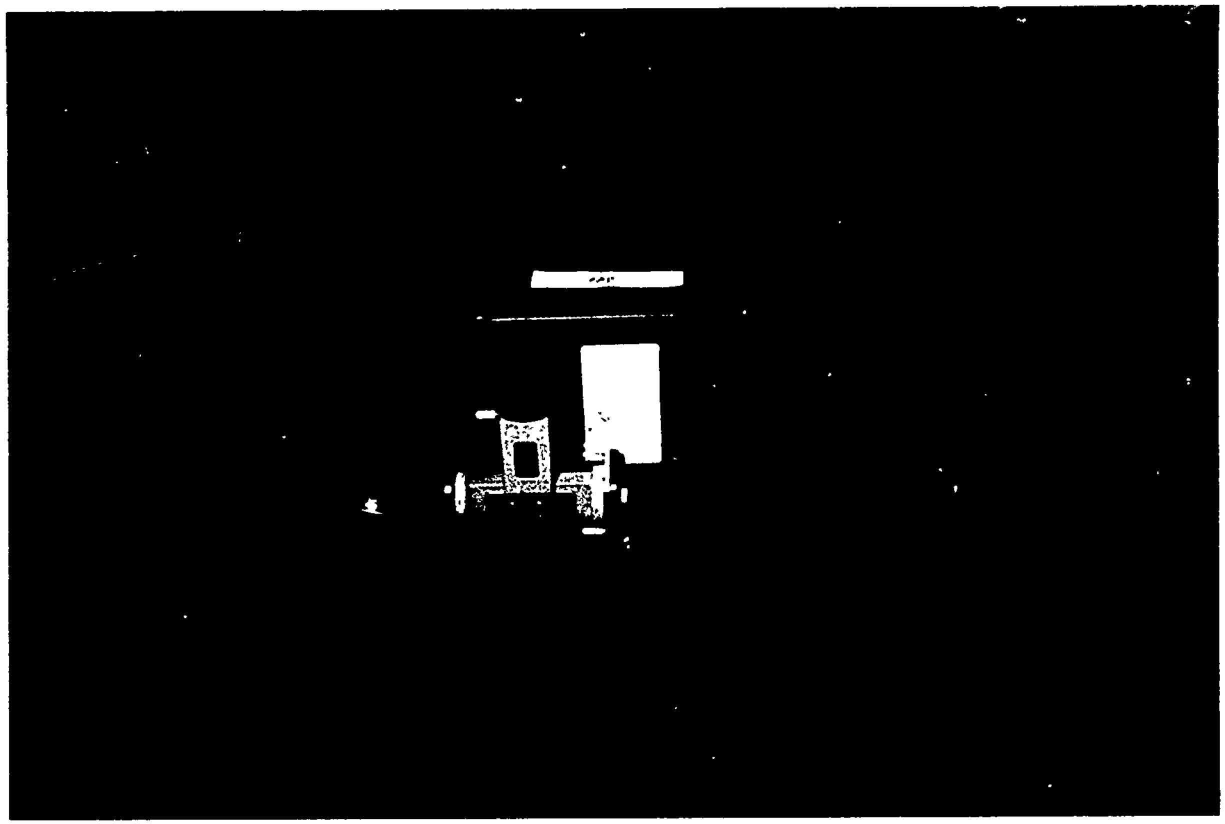

Fig 22. Jones Transmitter Showing Back of Phase Shifter

of the anechoic chamber. The receiver is mounted with its antennas centered in the open end of the anechoic chamber. Data is read from each of the six relative power indicators (HP 415) and recorded (Fig 23).

It may be desirable to orient the unknown antenna to put the major axis of its polarization ellipse parallel to vertical, horizontal, +45° or -45°. To accomplish this the unknown antenna is rotated on its longitudal axis until the particular meter indicates maximum reading which shows that the major axis is approximately aligned with that polarization. This practice is useful as a check on the repeatability of the measurements and to reveal any unwanted multipath reflections which would effect one polarization more then another.

The amplitude and phase settings for the Jones transmitter come from the system processor which would calculate them from the modified Stokes vector. The modified Stokes vector results from modulating the received Stokes vector with the desired program. The processor was not built for the experiment, however, a description of this processing procedure follows.

Processor Calculation

The six indicator readings are converted to relative power levels, for example -3.03 dB converts to .5 relative power. The received Stokes vector (absolute power is not measured, only coherent polarization components) is found by subtracting the measured data pairs as follows:

M = (Horizontal - vertical) power

C = (45°- (-45°)) power

S = (Left - Right circular) power

then

Fig 23. Relative Power Meters (below) Signal Source (above)

$$I = +\sqrt{M^2 + C^2 + S^2}$$

In general, I will differ from unity so the reciprocal of the calculated value of I is used to normalize each of the above Stokes vector elements.

The coherent Stokes vector is then:

$$\begin{bmatrix} (1) \\ (\frac{M}{I}) \\ (\frac{C}{I}) \\ (\frac{S}{I}) \end{bmatrix}$$

The polarization modulation, in the feasibility system concept, is impressed in the form of a Mueller matrix which pre-multiplies the received coherent Stokes vector. Then the modified (normalized) vector is

$$\begin{bmatrix} A & B & C & D \\ E & F & G & H \\ I & J & K & L \end{bmatrix} \times \begin{bmatrix} 1 \\ \frac{M}{I} \\ \frac{C}{I} \\ \frac{S}{I} \end{bmatrix} = \begin{bmatrix} 1 \\ M' \\ C' \\ S' \end{bmatrix}$$

To transmit the modifed polarization using the Jones transmitter it is necessary to change the above modified Stokes vector into a Jones vector. This proceeds as follows:

$$A_x = \sqrt{\frac{I + M}{2}}$$

$$A_y = \sqrt{\frac{I - M}{2}}$$

$$\cos \delta = \frac{C}{\sqrt{I^2 - M^2}}$$

$$\sin \delta = \frac{S}{\sqrt{I^2 - M^2}}$$

Using both of the latter two equations removes any quadrant ambiguity from δ. The magnitude of δ should be equal when calculated from the sin and cos term. Since any differences are due to errors, the two magnitudes should be averaged. Placing these elements in vector form yields

$$\begin{bmatrix} A_x \\ A_y e^{j\delta} \end{bmatrix}$$

which is the necessary Jones vector.

The amplitude difference in dB = $20 \log_{10} (\frac{A_x}{A_y})$ and the phase delay for the vertical channel is δ. These are programmed into the Jones transmitter as previously discussed.

Next actual measurement data is used to demonstrate the procedure. An "unknown" antenna was positioned with its major axis approximately vertical. The resulting data was:

Horizontal	-10 dB
Vertical	0 dB
45°	-1.4 dB
-45°	-4.2 dB
left circular	-0.7 dB
right circular	-6.2 dB

Converting the dB readings to relative power gives:

Horizontal	0.1
Vertical	1.0
45°	0.7244
-45°	0.3802
left circular	0.8511
right circular	0.2399

Subtracting the pairs and normalizing

$$M = -.9$$

$$C = 0.3442$$

$$S = 0.6113$$

$$I^2 = 1.302$$

$$I = 1.141$$

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} = \begin{bmatrix} 1 \\ -.7888 \\ 0.3017 \\ 0.5358 \end{bmatrix}$$

The modulation program requires orthogonal polarization to be sent back so the Mueller matrix multiplication is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -0.7888 \\ 0.3017 \\ 0.5358 \end{bmatrix} = \begin{bmatrix} 1 \\ +0.7888 \\ -0.3017 \\ -0.5358 \end{bmatrix} = \begin{bmatrix} I' \\ M' \\ C' \\ S' \end{bmatrix}$$

From the primed Stokes elements find

$$A_x = 0.9457$$

$$A_y = 0.3250$$

$$\left.\begin{array}{l} \delta = 119.4°,\ -119.4° \text{ from cos} \\ \delta = -60.6°,\ -119.4° \text{ from sin} \end{array}\right\} \delta = -119.4°$$

(which is the unambiguous solution)

VII Results

Data were measured using the equipment described in section III with the procedure of section VI. Polarization of the electromagnetic waves, created by four "unknown" antennas, was measured and the polarization ellipse was calculated. The Jones vector representation of the responding wave was then calculated assuming a modulation program requiring orthogonal response.

Data measured before the anechoic chamber became available is omitted from this section due to the likelihood of multipath reflections which would introduce errors. That data is, however, included in Appendix A. The four available "unknown" antennas are shown in Fig 20; they are:

(a) An I band, right circularly polarized, cavity backed spiral (RCP)

(b) An I band linearly polarized horn antenna

(c) An I band, left circularly polarized horn antenna (LCP)

(d) A left circularly polarized, cavity backed spiral antenna operated above its design frequency range in order to create a wave having a high axial ratio (This antenna is known as low band spiral)

These test antennas were individually positioned to radiate toward the Stokes receiver. The relative power level indicated at each of the six detector positions was recorded and used as input data for each calculation.

Data of 2 Oct 80 (in anechoic chamber)

RCP Spiral

H	-2.7	.5370	.1297	$( )^2 = .0168$
V	-3.9	.4074		
45°	-2.5	.5623	.1157	$( )^2 = .0134$
-45°	-3.5	.4467		
LCP	-26.6	.0022	-.9978	$( )^2 = .9956$
RCP	0	1.0000		

$$I^2 = .9956 + .0134 + .0168 = 1.026$$

$$I = 1.013$$

Normalized

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} = \begin{bmatrix} 1 \\ .1281 \\ .1142 \\ -.9852 \end{bmatrix}$$

$$\tan 2\tau = \frac{C}{M} = \frac{.1142}{.1281} = .8915$$

$$2\tau = 41.72°$$

$$= 20.8°$$

$$\sin 2\varepsilon = -9852$$

$$2\varepsilon = -80.1°$$

$$\varepsilon = -40°$$

$$AR = \cot 40° = 1.19 \qquad 1.51 \text{ dB}$$

RCP Spiral

Assuming Orthogonal Modulation Requirement

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ .1281 \\ .1142 \\ -.9852 \end{bmatrix} = \begin{bmatrix} 1 \\ -.1281 \\ -.1142 \\ +.9852 \end{bmatrix} = \begin{bmatrix} I' \\ M' \\ C' \\ S' \end{bmatrix}$$

$$A_x = \sqrt{\frac{I' + M'}{2}} = .66027$$

$$A_y = \sqrt{\frac{I' - M'}{2}} = .75103$$

$$\delta = \cos^{-1} \frac{C'}{\sqrt{I'^2 - M'^2}} = \cos^{-1} - .11515 = \pm 96.6°$$

$$\delta = \sin^{-1} \frac{S'}{\sqrt{I'^2 - M'^2}} = 83.4°, \ 96.6°$$

$\delta = +96.6°$ is the unambiguous solution

$$J = \begin{bmatrix} A_x \\ A_y e^{j\delta} \end{bmatrix} = \begin{bmatrix} .66027 \\ .75103e^{j96.6°} \end{bmatrix}$$

Data of 17 Oct 80

I Band Linear Horn

H	-2.7	.5754	.0743	$(\ )^2 = .0055$
V	-3	.5012		
45	0	1.000	.9975	$(\ )^2 = .9950$
-45	-26	.0025		
LCP	-3.1	.4898	.0220	$(\ )^2 = .0005$
RCP	-3.3	.4677		

$$I^2 = .0005 + .9950 + .0055 = 1.001$$

$$I = 1.0005$$

Normalized

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} = \begin{bmatrix} 1 \\ .07426 \\ .9970 \\ .0220 \end{bmatrix}$$

$$\tan 2\tau = \frac{C}{M} = \frac{.9970}{.07426} = 13.43$$

$$2\tau = 85.7°$$

$$\tau = 42.9°$$

$$\sin 2\varepsilon = .0220$$

$$2\varepsilon = 1.26°$$

$$\varepsilon = 0.6°$$

$$AR = \cot 0.6° = 90.9 \qquad 39.2 \text{ dB}$$

I Band Linear Horn

Assuming Orthogonal Modulation Requirement

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ .07426 \\ .9970 \\ .0220 \end{bmatrix} = \begin{bmatrix} 1 \\ -.07426 \\ -.9970 \\ -.0220 \end{bmatrix} = \begin{bmatrix} I' \\ M' \\ C' \\ S' \end{bmatrix}$$

$$A_x = \sqrt{\frac{I' + M'}{2}} = .6803$$

$$A_y = \sqrt{\frac{I' - M'}{2}} = .7329$$

$$\delta = \cos^{-1} \frac{C'}{\sqrt{I'^2 - M'^2}} = \pm 178.7°$$

$$\delta = \sin^{-1} \frac{S'}{\sqrt{I'^2 - M'^2}} = -1.3°, -178.7°$$

$\delta = -178.7°$ is the unambiguous solution

$$J = \begin{bmatrix} A_x \\ A_y e^{j\delta} \end{bmatrix} = \begin{bmatrix} .6803 \\ .7329e^{-j178.7°} \end{bmatrix}$$

Data of 2 Oct 80 (in anechoic chamber)

LCP horn max vert

-3.5	.4467	-.1422	$( )^2 = .0202$
-2.3	.5888		
-3.2	.4786	-.0342	$( )^2 = .0012$
-2.9	.5129		
0	1.000	.999	$( )^2 = .998$
-30	.001		

$$I^2 = .0202 + .0012 + .998 = 1.0193$$

$$I = 1.0096$$

Normalized

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} = \begin{bmatrix} 1 \\ -.1408 \\ -.0339 \\ .9894 \end{bmatrix}$$

$$\tan 2\tau = \frac{-.0339}{-.1408} = .2408$$

$$2\tau = 193.5$$

$$\tau = 96.8^\circ$$

$$\sin 2\varepsilon = .9894$$

$$2\varepsilon = 81.7^\circ$$

$$\varepsilon = 40.8^\circ$$

AR = 1.157 1.27 dB

LCP Horn

Assuming Orthogonal Modulation Requirement

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -.1408 \\ -.0339 \\ +.9894 \end{bmatrix} = \begin{bmatrix} 1 \\ .1408 \\ .0339 \\ -.9894 \end{bmatrix} = \begin{bmatrix} I' \\ M' \\ C' \\ S' \end{bmatrix}$$

$$A_x = \sqrt{\frac{I' + M'}{2}} = .75525$$

$$A_y = \sqrt{\frac{I' - M'}{2}} = .65544$$

$$\delta = \cos^{-1} \frac{C'}{\sqrt{I'^2 - M'^2}} = \pm 88.04$$

$$\delta = \sin^{-1} \frac{S'}{\sqrt{I'^2 - M'^2}} = 87.94,\ 92.06$$

$\delta = +88°$ is the unambiguous solution

$$J = \begin{bmatrix} A_x \\ A_y e^{j\delta} \end{bmatrix} = \begin{bmatrix} .7553 \\ .6554 e^{j88°} \end{bmatrix}$$

Data of 17 Oct 80

Lo Band Spiral

H	0	1.000		
			.8415	$(\ )^2 = .7081$
V	-8	.1585		
45	-4	.3981		
			-.1773	$(\ )^2 = .0314$
-45	-2.4	.5754		
LCP	-1.3	.7413		
			.4958	$(\ )^2 = .2459$
RCP	-6.1	.2455		

$$I^2 = .7081 + .0314 + .2459 = .9854$$

$$I = .9927$$

Normalized

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} = \begin{bmatrix} 1 \\ .8477 \\ -.1786 \\ .4994 \end{bmatrix}$$

$$\tan 2\tau = \frac{C}{M} = \frac{-.1786}{.8477} = -.2169$$

$$2\tau = -11.9°$$

$$\tau = -5.9°$$

$$\sin 2\varepsilon = \ 4994$$

$$2\varepsilon = 29.96°$$

$$\varepsilon = 15°$$

$$AR = \cot 15° = 3.74 \qquad 11.4\ \text{dB}$$

Lo Band Spiral

Assuming Orthogonal Modulation Requirement

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ .8477 \\ -.1786 \\ .4994 \end{bmatrix} = \begin{bmatrix} 1 \\ -.8477 \\ .1786 \\ .4994 \end{bmatrix} = \begin{bmatrix} I' \\ M' \\ C' \\ S' \end{bmatrix}$$

$$A_x = \sqrt{\frac{I' + M'}{2}} = .2759$$

$$A_y = \sqrt{\frac{I' - M'}{2}} = .9612$$

$$\delta = \cos^{-1} \frac{C'}{\sqrt{I'^2 - M'^2}} = \pm 70.3°$$

$$\delta = \sin \frac{S'}{\sqrt{I'^2 - M'^2}} = -70.3°, -109.7°$$

$\delta = -70.3°$ is the unambiguous solution

$$J = \begin{bmatrix} A_x \\ A_y e^{j\delta} \end{bmatrix} = \begin{bmatrix} .2759 \\ .9612 e^{-j70.3°} \end{bmatrix}$$

VIII Conclusions & Recommendations

Conclusions

From the polarization methods studied, a responsive system realization has been formulated. The receiver uses two orthogonally polarized antennas to feed a circuit containing hybrid Tees. Stokes parameters are obtained by adding or subtracting selected pairs of six detected outputs.

A processor (which was not built nor tested) changes the incoming polarization state, via Mueller matrix multiplications, to the new state prescribed by the modulation program. This results in complete flexibility of polarization response to any arbitrary input signal. The processor generates control signals, in Jones vector format, which control the amplitude and phase (relative) of the signals transmitted by two orthogonally polarized transmit antennas. The transmit antennas have phase centers approximately coincident with the phase centers of the receive antennas to provide off boresight polarization response accuracy.

Throughout the study emphasis was placed on coherently polarized signals and rapid response time. Coherently polarized signals are typically associated with radars or communication systems while unpolarized signals are found in radio astronomy work.

Recommendations

Future study efforts could include the following investigations:

(1) Design, build and bench test the processor connected to the experimental microwave setup of this thesis.

(2) Investigate the effects of positioning the phase centers of the receive antennas different from those of the transmit antennas.

(3) Investigate air vehicle installation constraints such as radomes, broadband frequency coverage requirements, multipath interference caused by skin reflections from the vehicle and integration of the polarization responsive system with antennas that must cover large spatial volumes or must beam steer.

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APPENDIX A

RAW DATA

Sheet 1

Date 1980	Unknown Antenna	Decibels (negative)						Remarks
		H	V	45	-45	LCP	RCP	
Sep 2	Lo Band LCP Spiral	0	13.5	3.8	3.2	1.2	5	
Sep 2	RCP Spiral	1.3	4.5	2.5	4	0	14	
Sep 2	LCP Horn	2.3	5	3	3.6	18.5	0	
Sep 4	Lo Band LCP Spiral	0	16.5	5	3.[illegible]	1.6	3.9	
Sep 4	RCP Spiral	2.6	3.7	3.2	5	18.2	0	
Sep 4	Linear Horn	0	20	4	4	2.4	2.6	
Sep 4	Linear Horn	20	0	2.6	2.6	2.8	2.9	
Sep 10	RCP Spiral	2.3	3.7	2.5	4.5	16	0	
Sep 10	RCP Spiral	3.3	2.9	4.5	1.5	17	0	Improved Setup with Hybrid Ts
Sep 10	Lo Band LCP Spiral	6.6	0.7	0	9	0.8	7	
Sep 10	Lo Band LCP Spiral	0	8	2	2.2	1.1	5.7	
Sep 10	Lo Band LCP Spiral	11.5	0	1.5	4.6	1.4	5.7	
Sep 10	Linear Horn	30	0	3	2.9	2.9	3.1	
Sep 10	Linear Horn	0	30	2.8	2.6	4	2.9	
Sep 10	LCP Horn	2.2	2.5	1	4	0	17	

Table 1. Raw Data

Sheet 2

Date 1980	Unknown Antenna	Decibels (negative)						Remarks
		H	V	45	-45	LCP	RCP	
Sep 26	RCP Spiral	3.4	3	3.3	2.8	18.5	0	
Sep 26	Linear Horn	0	>15	3.2	3.4	3.3	3	
Sep 26	Linear Horn	-14.5	0	3.5	3.0	3.1	3.2	
Sep 26	Linear Horn	3.2	2.9	16	0	3	3.4	
Sep 26	Linear Horn	3.2	3.2	0.9	15	3.5	2.8	
Sep 26	Lo Band LCP Spiral	0	8	0.7	4.8	0.6	5.2	
Sep 26	Lo Band LCP Spiral	10	0	1.4	4.2	0.7	5.2	
Sep 26	Lo Band LCP Spiral	3.3	1.5	0	12	0.5	6.6	
Sep 26	Lo Band LCP Spiral	2.5	2.7	7.5	0	0.7	5.5	
Sep 26	LCP Horn	2.8	3.3	2.3	4.2	0	24	
Sep 26	LCP Horn	3.6	2.2	2.1	4.1	0	>25	
Sep 26	Linear Horn	14	0	3.3	3.1	3.3	2.8	
Sep 26	Linear Horn	0	15	3.2	3.2	3.4	2.5	
Sep 26	RCP Spiral	3	3.4	3.3	2.8	18.5	0	
Oct 2	RCP Spiral	2.7	3.9	2.5	3.5	26.6	0	in anechoic chamber
Oct 2	LCP Horn	3.5	2.3	3.2	2.9	0	30	
Oct 3	RCP Spiral	2.9	3.1	2.2	4.3	21	0	
Oct 3	RCP Spiral	3.7	2.3	2.3	4.3	19.5	0	

Table II. Raw Data

Sheet 3

Date 1980	Unknown Antenna	Decibel (negative)						Remarks
		H	V	45	-45	LCP	RCP	
Oct 17	RCP Spiral	3	3.1	2.3	3.5	25.7	0	in anechoic chamber
Oct 17	RCP Spiral	3.1	3.2	1.8	4.4	18.7	0	
Oct 17	RCP Spiral	3.8	2.4	2	4	20.8	0	
Oct 17	LCP Horn	0.9	3.7	2.6	3 .	0	15	
Oct 17	LCP Horn	1.8	2.7	3.8	2.3	0	23	
Oct 17	Lo Band LCP Spiral	0	8	4	2.4	1.3	6.1	
Oct 17	Lo Band LCP Spiral	6	0	1.2	4.1	0.3	7.3	
Oct 17	Linear Horn	0	18	3.7	3.7	3.6	4	
Oct 17	Linear Horn	>18	0	3	3.1	3.1	3.2	
Oct 17	Linear Horn	2.4	3	0	26	3.1	3.3	
Oct 17	Linear Horn	2	3	30	0	2.9	3.1	

Table III. Raw Data

Appendix B

Repeatability Measurements

A set of 10 measurements of each Stokes parameter was made to determine the repeatability of the experiment. During the early part of the experiment it was found necessary to select the six indicating meters for their 1000 Hz bandpass characteristics to coincide. These filters are quite narrow so the effect of one meter having its filter slightly off the modulation frequency (1000 Hz) is operation on the steep slope of its bandpass characteristic, where any slight change in modulating frequency results in a change in meter reading.

Once the meters were selected, the experiment became quite stable and repeatable as the following data shows. Gaussian distribution is assumed. The measurements are tabulated in Table IV.

Calculation

Converting the mean Stokes parameters to equivalent Jones vector representation (by the methods described earlier), then varying the Stokes parameters by the measured standard deviation yields the amplitude (A_x and A_y) errors and the phase (δ) error associated with the experiment. The mean Stokes vector was found to be

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} = \begin{bmatrix} 1.08394 \\ -1.02473 \\ .91439 \\ 0.57552 \end{bmatrix}$$

which converts to

$$\begin{bmatrix} 0.75576 \\ 0.73500e^{j27.2°} \end{bmatrix}$$

Therefore the error in A_x = 3.85%, the error in A = -1.28%, and the phase error = -15.8%.

The repeatability errors in the experimental setup have been shown to cause less than .33 dB in amplitude of either the vertical or horizontal channel and 6 degrees of phase for the case tested.

Table IV

Repeatability Data in DB

Trial	H	V	45	-45	LCP	RCP
1	-3.2	0	0	-11.4	-1.6	-5.7
2	-2.9	-2.3	0	-11.5	-1.2	-5.6
3	-3	-2.3	0	-11.3	-0.8	-5.7
4	-2.6	-2.6	0	-10.7	-0.6	-5.9
5	-2.8	-2.1	0	-10.2	-0.5	-6.1
6	-2.5	-2.4	0	-10.3	-0.9	-6.1
7	-2.2	-2.8	0	-10.3	-0.5	-6.0
8	-2.3	-2.5	0	-10.6	-0.7	-6.0
9	-2.4	-2.6	0	-10.3	-0.6	-5.9
10	-2.7	-2.5	0	-10.4	-0.7	-6.0

Table V

Repeatability Data in Stokes Parameters

Trial	I	M	C	S
1	1.0228	-.0837	+.9276	+.4227
2	1.0501	-.0760	.9292	+.4832
3	1.0869	-.08766	+.9259	.5626
4	1.1018	0	+.9149	.6139
5	1.1151	-.0918	+.9045	.6458
6	1.0696	-.0131	+.9067	.5673
7	1.1126	+.0778	+.9067	.6401
8	1.0927	+.02650	+.9129	.5999
9	1.0986	+.0259	.9067	.6198
10	1.0892	-.0253	.9088	.5999
	m=1.08394	m=-.024736	m=.91439	m=.57552
	σ^2=.000753	σ^2=.003101	σ^2=.0000835	σ^2=.004584
	σ=.02744	σ=.05569	σ=.00914	σ=.06771

Appendix C

Error Analysis

To gain insight into the accuracy requirements of a polarization responsible system, an error calculation was performed. The analysis shows the effect of four cases of assumed amplitude and phase error combined with assumed processor error. The usual technique of error analysis calculates the partial derivatives of the measured quantities with respect to the error-inducing quantity; however, due to the complex relationships involved in the polarization matrix manipulations, that approach was abandoned.

The error in creating the desired polarization response can occur in the three parameters A_x, A_y, and δ of the Jones vector representation or any of the four Stokes vector elements.

The system selected for the experiment of this thesis was used as the model for this investigation. It uses two channels, the vertical and the horizontal, which ideally maintain phase track throughout the system (except for the intentional 90° phase shift in the circularly polarized receiver). As a consequence of this configuration, amplitude and phase were selected as the parameters to use to investigate the effect of system errors.

Starting with the Jones vector representation

$$\begin{bmatrix} A_x \\ A_y e^{j\delta} \end{bmatrix}$$

the amplitudes A_x and A_y can be associated with the gain or loss in the horizontal and vertical channel of the receiver and δ can be associated with the phase difference between the two channels. The effect of amplitude or phase error will be investigated by taking four cases of assumed error. The first case is where the total amplitude imbalance is $\pm$ 4 percent (.35 dB) per channel, the phase error is 1 degree, and the processor contributes 2 percent error to each Stokes element. The second case is where the amplitude imbalance is 10 percent (.92 dB), the phase error is 2 degrees and the processor contributes 10 percent. The third case is where the amplitude's imbalance is 20 percent (1.9 dB), the phase error is 10 degrees, and the processor contributes 10 percent. Case IV is the same as Case III except the processor errors are reversed in terms of which Stokes element increase and which decreases in magnitude. These cases typify the achievable range of errors in a well designed, flyable system.

First the reflection of these errors or the Stokes parameters will be calculated individually. Next, processor errors will be included. They are assumed to cause a fixed percent error in each Stokes parameter, increasing the magnitude of the M and S elements, and decreasing the magnitude of the I and C elements. Then the new Stokes vector is calculated, assuming that orthogonal polarization response is desired. Finally, the output Jones vector (with error contributions) is calculated for comparison with the error-free case.

<u>Calculation</u>

Let the input Jones vector be represented as

$$\begin{bmatrix} A_x \\ A_y e^{j\delta} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1e^{j45°} \end{bmatrix}$$

in the error-free case. After including the assumed errors, the Stokes parameters are calculated from

$$I = A_x^{\ 2} + A_Y^{\ 2}$$

$$M = A_x^{\ 2} - A_Y^{\ 2}$$

$$C = 2A_xA_Y \cos\ \delta$$

$$S = 2A_xA_Y \sin\ \delta$$

The processor errors are included as a multiplying factor on the magnitude of each Stokes element. Then the assumed orthogonality reverses the sign of the M,C,S values. Finally the new Jones vector is calculated through

$$A_x = \sqrt{\frac{I + M}{2}}$$

$$A_Y = \sqrt{\frac{I - M}{2}}$$

$$\delta = \cos^{-1} \frac{C}{\sqrt{I^2 - M^2}}$$

$$\delta = \sin^{-1} \frac{S}{\sqrt{I^2 - M^2}}$$

Results of the calculation are given in Table VI.

The investigation has shown a general trend of increasing error as the system is allowed to become less precise. There are exceptions to this general trend which occur, due to the error-cancelling properties of the system. As a general conclusion, the system accuracy should approach that of case I if errors larger than about 5 perc[illegible] are to be avoided.

Table VI. Error Data Summary

Parameter	Error-Free Case	Case I	Case II	Case III	Case IV
A_x	1	.96	.9	.8	.8
A_y	1	1.04	1.1	1.2	1.2
δ	45°	46°	47°	55°	55°
I	2	2.0032	2.02	2.08	2.08
M	0	-0.16	-0.4	-0.8	-0.8
C	$2/\sqrt{2}$	1.3871	1.350	1.101	1.101
S	$2/\sqrt{2}$	1.4364	1.448	1.573	1.573
processor error					
I	2	2.0432	2.222	2.288	1.872
M	0	-.1568	-0.36	-0.72	-1.88
C	$2/\sqrt{2}$	1.4148	1.485	1.211	+.9909
S	$2/\sqrt{2}$	1.4077	1.303	1.416	+1.7303
I'	2	2.0432	2.222	2.288	1.872
M'	0	+.1568	+0.36	+0.72	+.88
C'	$-2\sqrt{2}$	-1.4148	-1.485	-1.211	-.9909
S'	$-2\sqrt{2}$	-1.4077	-1.303	-1.416	-1.7303
A_x'	1.00	1.0488	1.1362	1.2264	1.1730
A_y'	1.00	.97119	0.9649	.8854	.7043
δ'	-135°	-133.9° or -136.3°	-132.6° or -143.5°	-123.9° or -139.3°	-126.9° or -90°
Error A_x	0	4.9%	13.6%	22.6%	17.3%
Error A_y	0	-2.9%	-3.5%	-11.5%	29.6%
Error δ (maximum)	0	-1.1%	-6.3%	-8.2%	33.3%

Appendix D

The computer listing of a program to premultiply a Stokes or Jones vector by a Mueller or Jones matrix.

```
      PROGRAM POLARIZ(INPUT,OUTPUT,
      TAPE5=INPUT,TAPE6=OUTPUT)
C     PROGRAM TO MULTIPLY MULTIPLE MATRICES
C     THE NUMBER OF ROWS (L) AND THE NUMBER
      OF COLUMNS (M) CANNOT EXCEED
C     FIVE AND FIVE RESPECTIVELY
C     THE NUMBER OF ROWS (N) AND THE NUMBER
      OF COLUMNS (L) CANNOT EXCEED
C     FIVE AND FIVE RESPECTIVELY
        COMPLEX A,B,C
        DIMENSION A(5,5)
        DIMENSION B(5,5)
        DIMENSION C(5,5)
C     READ INPUT DATA
        READ(5,*) L,M,N
        IF (N.EQ.0) GO TO 500
        PRINT*,"L=      ",L
        PRINT*,"M=      ",M
        PRINT*,"N=      ",N
C     FILL THE B MATRIX
        READ(5,*)((B(K,I),I=1,M),K=1,L)
        PRINT*,"   MATRIX B"
        DO 5 K=1,L
5       WRITE(6, * )(B(K,I),I=1,M)
C     FILL THE A MATRIX
15      READ(5,*)((A(K,I),I=1,L),K=1,N)
        IF (EOF(5LINPUT).NE.0) GO TO 205
        PRINT*,"   MATRIX A"
        DO 20 K=1,N
20      WRITE(6, * )(A(K,I),I=1,L)
125     CONTINUE
C     COMPUTE THE C MATRIX
        DO 140 K=1,N
        DO 140 I=1,M
        C(K,I)=0
        DO 140 J=1,L
140     C(K,I)=C(K,I)+A(K,J)*B(J,I)
        DO 160 J=1,M
        DO 150 I=1,N
150     B(I,J)=C(I,J)
160     CONTINUE
C     GO FOR ANOTHER A MATRIX
        GO TO 15
205     PRINT*,"   MATRIX C"
        DO 210 I=1,N
210     WRITE(6, * )(C(I,J),J=1,M)
500     STOP "END OF PROGRAM"
        END
```

Appendix E

Jones Matrix to Mueller Matrix Conversion

and

Jones Vector to Stokes Vector Conversion

Jones to Mueller Conversion (Matrices)

Jones matrices cannot in general be derived from Mueller matrices but Mueller matrices can be derived from Jones matrices in much the same way that power can be derived from complex field components. (Such derivation is given by Schmeider in Journal of Optical Society of America, Volume 59, Number 3, March 1959, "Stokes-Algebra Formalism".)

Let

$$A = \text{Jones matrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$M = \text{Mueller matrix} =$$

$$\begin{bmatrix} 1/2(a^2+b^2+c^2+d^2) & Re(a^*b+c^*d) & jIm(a^*b+c^*d) & 1/2(a^2-b^2+c^2-d^2) \\ Re(a^*c+b^*d) & Re(a^*d+b^*c) & jIm(a^*d-b^*c) & Re(a^*c-b^*d) \\ -jIm(a^*c+b^*d) & -jIm(a^*d+b^*c) & Re(a^*d-b^*c) & -jIm(a^*c-b^*d) \\ 1/2(a^2+b^2-c^2-d^2) & Re(a^*b-c^*d) & jIm(a^*b-c^*d) & 1/2(a^2-b^2-c^2+d^2) \end{bmatrix}$$

also

$$M_{00} + M_{11} + M_{22} + M_{33} = T_r\ \{A^{\dagger}\}\ T_r\ \{A\}$$

$A^{\dagger}$ is inverse of A

Jones Vector to Stokes Vector Conversion

The Jones vector is in general not obtainable from the generalized Stokes vector, however, if we limit to the case of fully polarized waves (d = 1) then the conversion is accomplished through the following equations (Gerrard and Burch, 1975)

$$A_x = \sqrt{\frac{I + M}{2}}$$

$$A_y = \sqrt{\frac{I - M}{2}}$$

$$\cos \delta = \frac{C}{\sqrt{I^2 - M^2}}$$

$$\sin \delta = \frac{S}{\sqrt{I^2 - M^2}}$$

where the Stokes vector is

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix}$$

and the Jones vector is

$$\begin{bmatrix} A_x \\ A_y e^{j\delta} \end{bmatrix}$$

If instead the Jones vector is known, then the Stokes vector can be found from

$$I = \sqrt{A_x^2 + A_Y^2}$$

$$M = \sqrt{A_x^2 - A_Y^2}$$

$$C = 2A_xA_Y \cos \delta$$

$$S = 2A_xA_Y \sin \delta$$

Vita

Warren R. Minnick was born on 25 March 1929 in Atlanta, Georgia. He graduated from high school in Columbus, Ohio and attended The Ohio State University, from which he received the degree of Bachelor of Electrical Engineering in June 1964. He was employed by North American Aviation at the time of graduation and remained there until 1968, when he entered civil service at Wright-Patterson Air Force Base, Ohio. As a project engineer for the Air Force he supported electronic countermeasures development programs for the B-58, F-15, EF-111 and other special-purpose aircraft. In October 1979 he entered the School of Engineering, Air Force Institute of Technology, where he is working toward a Master's degree in electrical engineering.

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Responsive polarization methods and concepts were studied in terms of general system requirements. A primary goal was fast response. Matrix methods were found appropriate for describing the polarization states and modifications thereto.

A 9.3 GHz experiment, of one promising concept, was constructed. Stokes parameters were measured, modified and converted to Jones parameters for response.

Basic feasibility was established for the hybrid T circuit arrangement

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